APPLICATION OF COST EFFECTIVENESS TECHNIQUES TO SELECTION OF PREFERRED WARSHIP CHARACTERISTICS

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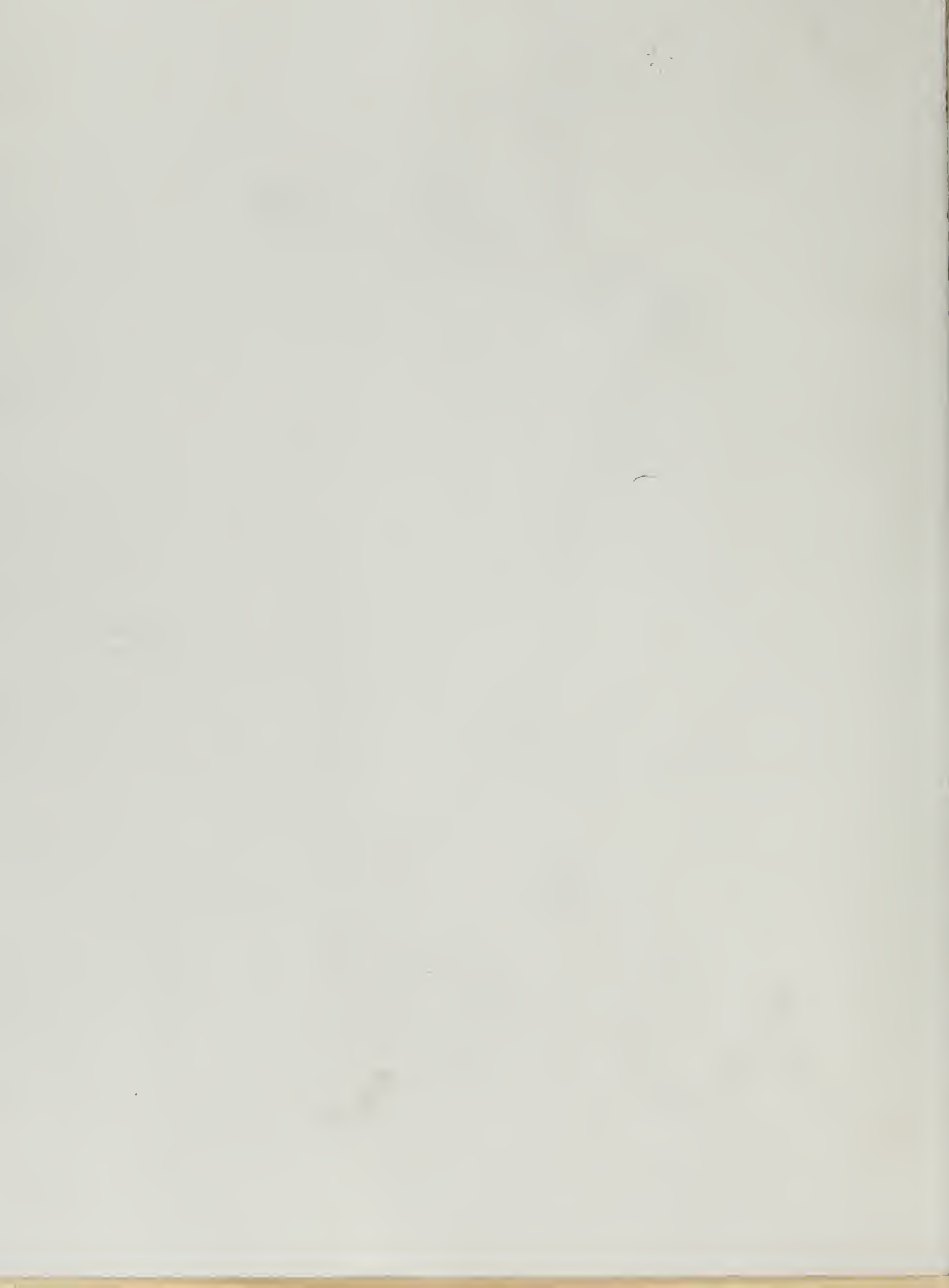# SELECTION OF PREFERRED WARSHIP CHARACTERISTICS

by

Louis K. McMillan, Jr.

This work is accepted as fulfilling the

thesis requirements for the degree of

MASTER OF SCIENCE

from the

United States Naval Postgraduate School

ABSTRACT

This paper discusses the applicability of cost effectiveness

methods to the problem of determining preferred design characteristics

of surface, anti-submarine warships. A short introduction to the concept

of cost effectiveness as applied to military weapons systems is followed

by a description of the methodology applicable to adapting cost effective-

ness techniques to selection of preferred warship design characteristics.

The surface anti-submarine vessel is used as a vehicle for adapting the

cost effectiveness methodology; explanations as to how the cost effective-

ness model may be expanded to include other types of surface ships is

included.

PREFACE

Since World War II the cost and complexity of military weapons systems has greatly increased. As a result there has been an increased desire by persons involved in defense planning to find a means of observing in quantitative terms the interrelationships between cost and effectiveness of military weapons systems. The cost effectiveness study is a tool designed to permit such observations. Cost effectiveness techniques have been successfully applied to air-borne weapons systems for some time. Air-borne weapons systems are relatively short lived systems, and are usually designed to perform one, or at most a few, military missions. To date there has been little effort toward applying cost effectiveness methods to optimization of ship-borne weapons systems. Because of their longer life and greater mission complexity, ship-borne weapons systems do not lend themselves so readily to study by cost effectiveness methods.

This paper is concerned with applying cost effectiveness methods to the selection of preferred design characteristics of surface ships employed in anti-submarine warfare. Chapter I is an introduction to cost effectiveness as an aid to selection of preferred military weapons systems. Chapter II outlines the methodology to be used in applying cost effectiveness techniques to the selection of preferred warship design characteristics, and structures the total cost equation in terms

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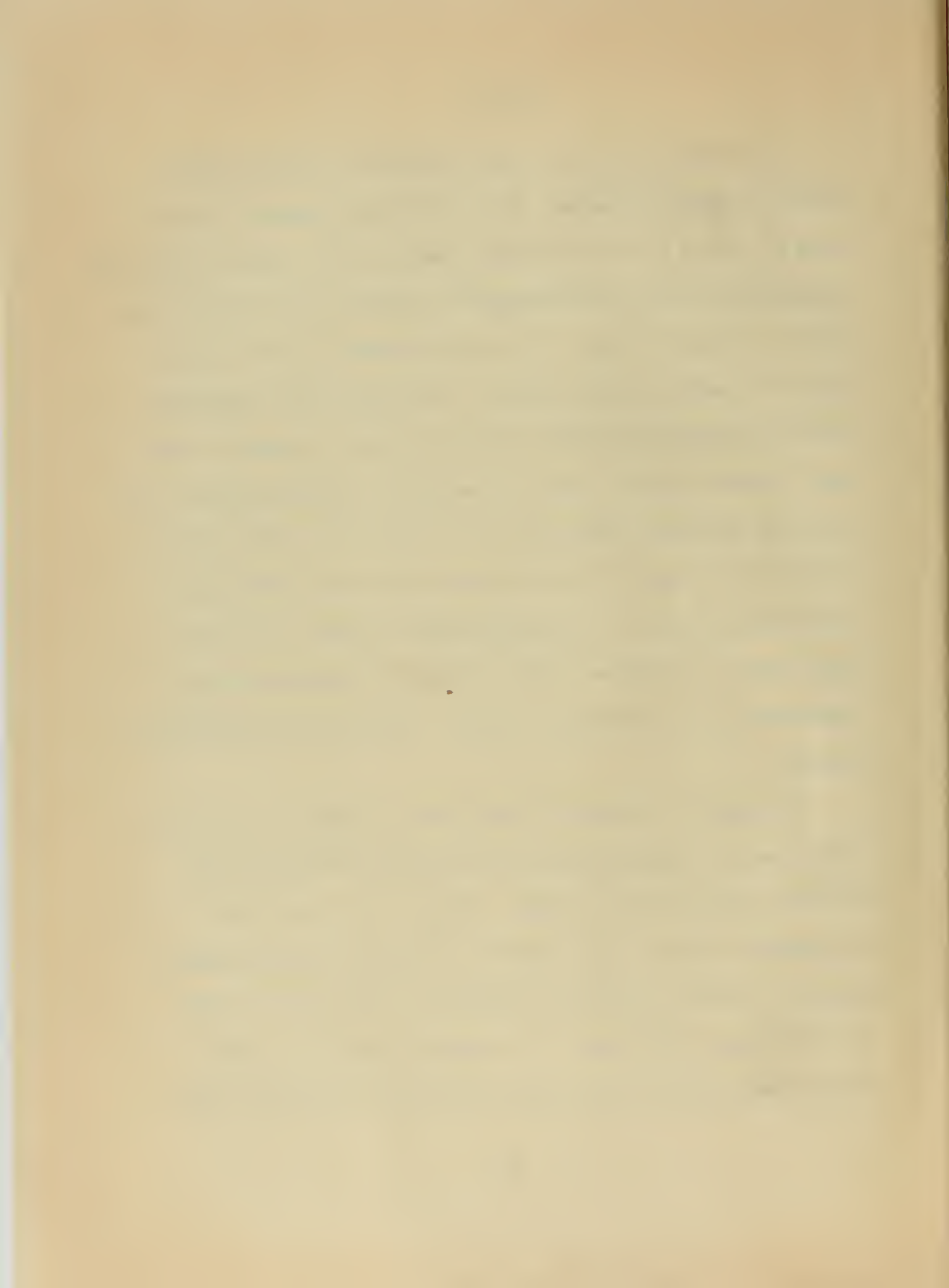
of major system cost divisions. In the case of anti-submarine warships

these divisions are; cost of ship procurement, ship operating costs,

initial cost of support facilities, and operating cost of support facilities.

Cost of ship procurement and ship operating costs are further reduced to

their component cost elements in Chapter II, and a short discussion of

the problems related to determining what support facility costs should be

apportioned to weapons systems using the facilities is included. In

Chapter III the various cost elements defined in Chapter II are presented

as functions of physical characteristics of the anti-submarine, surface

ship weapons system. Chapter IV considers the assembly of the total

cost equation and illustrates how the mathematical representation of

an assigned mission of a weapons system is formulated for use in a cost

effectiveness model. Comments and conclusions are contained in Chapter V.

The mathematical relationships of the cost elements to the physical

characteristics of the weapons system outlined in Chapter III have, in all

cases possible, been verified either by reference to authoritative works

on the subject of ship design and construction, or by curve fitting methods

to empirical data obtained from unclassified documents, particularly

reference, $\overline{/\underline{5}/}$, of the bibliography. In some cases the author was unable

to find sufficient data to substantiate mathematical relationships between

the cost of a system element and the physical characteristics of the

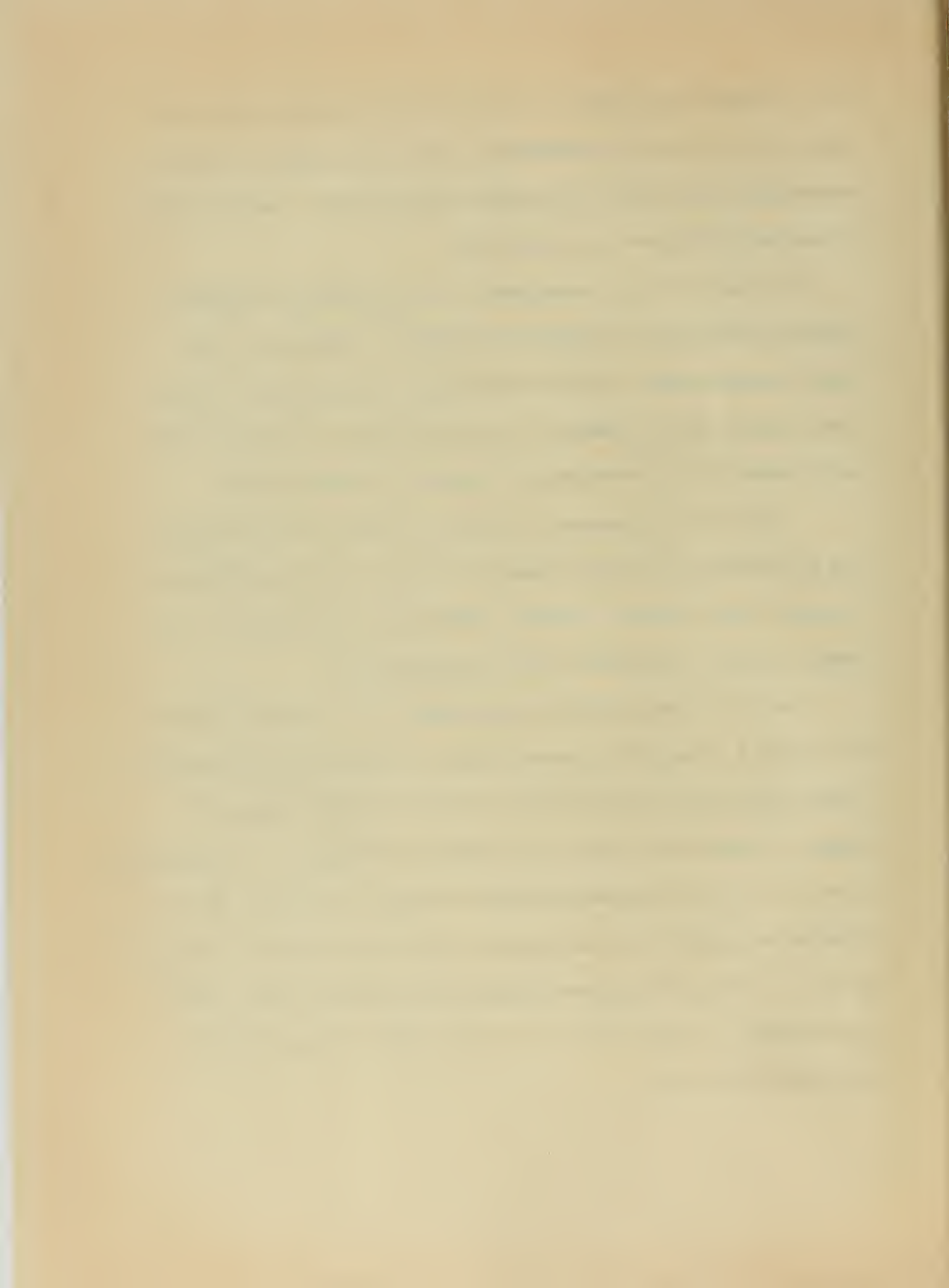weapons system. In those cases the author has made what seem to him
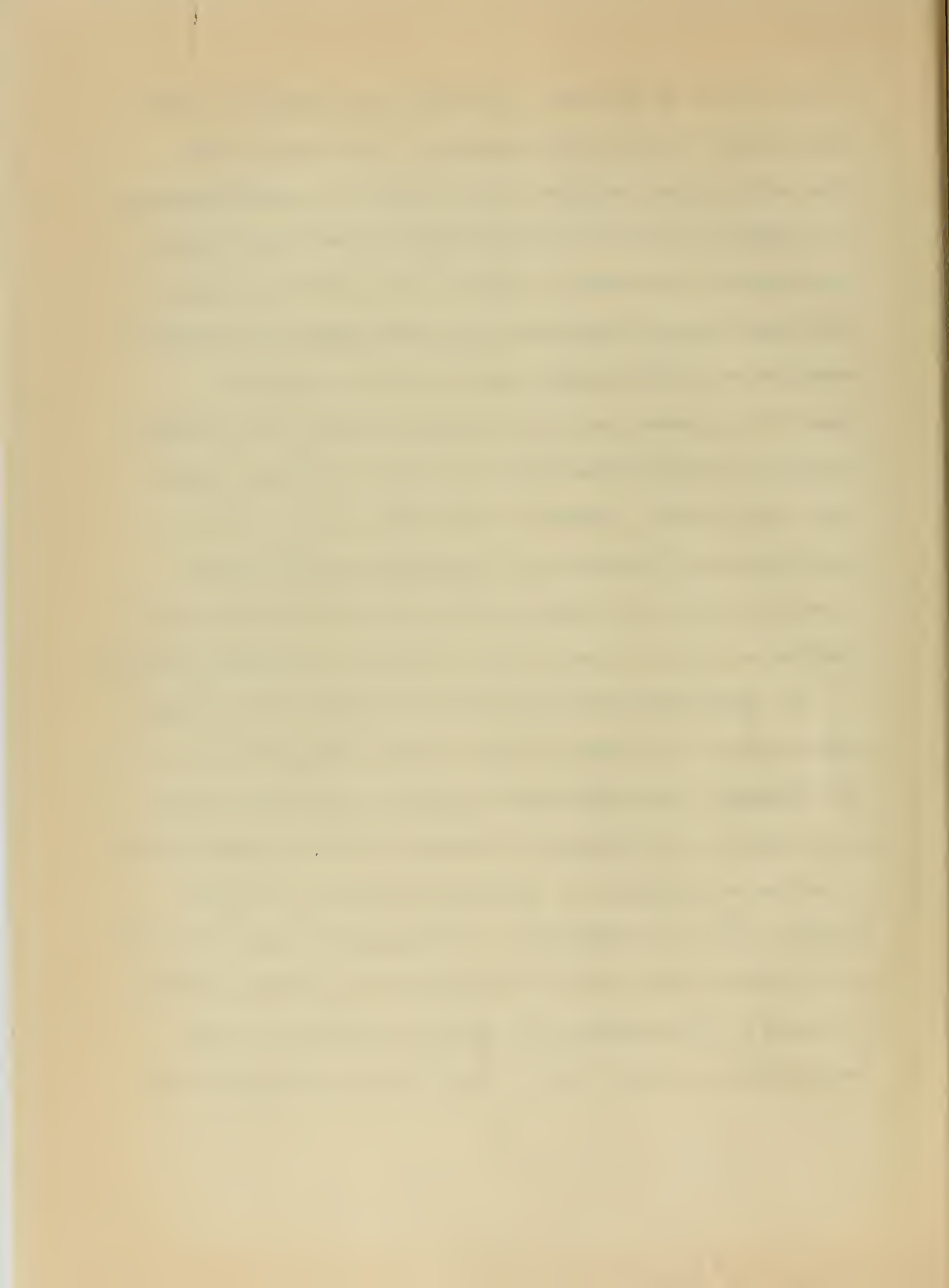
to be reasonable assumptions as to the mathematical form of the cost relationships, based on conversations and correspondence with persons experienced in the field of ship construction and design, and on a rather limited personal experience in this field.

As a result of this investigation it is concluded that cost effectiveness techniques are applicable to problems of selection of ship design characteristics, provided the mission structure used in the cost effectiveness model is adequate to cover the spectrum of missions that might reasonably be performed by a ship-borne weapons system.

I first became interested in this project while on a summer field trip at the Technical Military Planning Operation of the General Electric Company, Santa Barbara, California, during the summer of 1961. This paper is largely an extension of the work begun there.

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(Listed in the order of their use in the text)

C_T	Total weapons system cost
N	Number of ships required to produce a preset degree of effectiveness
Y	Time period over which the weapons system is to be considered
C_{sp}	Cost of individual ship procurement
C_{os}	Annual cost of operating one ship
C_{if}	Initial cost of support facilities required for one ship
C_{of}	Annual operating cost of support facilities required for one ship
C_h	Procurement cost of hull
C_p	Procurement cost of propulsion
C_{aa}	Procurement cost of anti-aircraft armament
C_{asw}	Procurement cost of anti-submarine armament
C_c	Procurement cost of communications equipment
C_{su}	Procurement cost of underwater search equipment
A_{sa}	Procurement cost of above water search equipment
C_f	Annual cost of fuel per ship
C_m	Annual cost of maintenance per ship
C_{cs}	Annual cost of consumable supplies per ship
C_{pe}	Annual cost of personnel per ship
$\triangle$	Displacement
L	Ship length

SHP	Shaft horsepower developed by the propulsion equipment of a ship
V	Maximum designed speed of a ship
P_1	Unit procurement cost of anti-submarine armament
P_2	Unit procurement cost of anti-aircraft armament
N_{asw}	Number of anti-submarine armament units installed
N_{aa}	Number of anti-aircraft armament units installed
R_u	Underwater detection range
R_a	Above water detection range
h	Vertical dimension of sonar transducer
SHP_c	The shaft horsepower utilized by a ship while proceeding at cruising speed
v_e	Cruising speed of ASW vessel
A	Patrol area to be covered in mission "A"
T	Revisit time at any point within area A
E	Endurance of ASW vessel unsupported
R_1	Repair and replenishment time for unsupported ASW vessel
N_1	The number of ASW vessels required to be on station continuously in order to accomplish the assigned mission
N_2	The number of ASW vessels required to keep one vessel on station at all times
d	Either distance to and from the patrol area, or convoy route distance depending on the context in which used
v_c	Convoy speed

W	Width of convoy
P	Anti-submarine vessel kill probability against a submarine
E_r	Repair limited endurance of an ASW vessel; i.e., the average number of days the vessel can operate away from shore based support facilities when stores, ammunition, and fuel are replenished underway from supply ships
R_2	Repair and replenishment time at a shore facility for a supported ASW vessel
N_3	The number of underway replenishments required by an ASW vessel during one repair limited cycle of $E + R_2$ days.
N_4	The number of underway replenishments that can be effected by one supply ship during one resupply mission
N_5	The number of replenishments at sea that can be effected by a supply ship during one repair limited cycle of an ASW vessel
R_3	Average underway replenishment time
R_4	Repair and replenishment time for supply ship
l	Average distance between replenishment rendezvous points
v_t	Supply ship cruising speed

CHAPTER I

THE COST EFFECTIVENESS STUDY AS A MILITARY PLANNING TOOL

1.   Philosophy of cost effectiveness

Most military operations analysis studies present their results in terms of measures of effectiveness. These measures of effectiveness are usually designed to provide a measure as to how well a particular weapons system or tactic will perform a specified task. Some well known examples of measures of effectiveness are probability of target destruction, expected bomb damage, and expected number of enemy personnel killed. Much has been said in current operations analysis literature on the pitfalls and problems of selecting adequate and representative measures of effectiveness $\sqrt{1}$, $\sqrt{2}$[1]. Recently, the idea of adding another dimension to operations research studies in the form of cost effectiveness has received a good deal of attention, and has begun to figure more and more prominently in defense planning. Cost, used in this sense, is not, strictly speaking, a measure of effectiveness, but represents another value continuum which may be used in conjunction with conventional measures of effectiveness. Cost often acts as a constraint which bounds an area of acceptable solutions. Within this area of acceptable costs the military planner might well be interested in using operations analysis techniques to determine which weapons system under consideration would produce the greatest return in terms of the selected measures of effectiveness per unit of cost.

[1]Numbers in brackets refer to bibliography on page 48.

The recently increased interest in cost effectiveness studies can generally be attributed to the continuing high level of military readiness the United States has been compelled to maintain since the advent of the cold war. The vastly increased cost and complexity of today's military weapons systems has prompted military planners to search earnestly for more precise means of evaluating and comparing proposed weapons systems before vast sums of money are spent on their procurement. National policy planners have been continually faced with the conflicting requirement of providing the United States with an adequate military posture and, at the same time, not sapping the nation's economy with crushing military expenditures to the extent that its vitality and natural capacity for expansion are destroyed. This all reduces to the problem of, what is the best way to allocate the country's limited resources? In this environment the cost effectiveness type of operations research study has been recognized as a valuable aid to certain facets of defense planning since it takes into account not only the military effectiveness of a planned weapons system, but also evaluates the efficiency of the system in terms of its cost. For a more comprehensive study of the problems connected with the allocation of resources to national defense the reader is referred to Hitch and McKean $\boxed{3}$.

2.   Defining a weapons system

The term "weapons system" as used in cost effectiveness studies is generally defined to mean the collection of equipage and operating

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personnel required to carry out a specified portion of a military mission. ——
Obviously, this definition leaves considerable leeway as to its inter-
pretation, and much must be drawn from the context in which the term
is used. Whereas one author might consider a particular type of missile
battery installed on board a warship as a weapons system, another might
consider the warship itself as the basic weapons system and the particular
missile battery as a subsystem. The warship, in turn, may be considered
as a part, or subsystem, of a still larger weapons system which includes
all types of surface warships. It follows, therefore, that it is necessary
to specify at the outset of such a study exactly what is considered as the
basic weapons system in order to avoid confusion.

The basic weapons system considered in this paper is a surface,
anti-submarine warship. This basic weapons system will include as
subsystems the installed armament, communications equipment, search
equipment, propulsion machinery, and operating personnel.

3. Units of cost measure

A natural question that arises at this point is how is the cost to
be measured? The most commonly used measure of cost is monetary.
However, it is apparent after some thought that there are certain cases
where the monetary cost of a military system is an inadequate and mis-
leading representation of the actual cost in resources incurred by the
system. An example of such a situation would be the case where one
wished to consider the cost of a certain system which required highly

skilled workers for construction of its components. If, as often is the case, the supply of these skilled workers is limited, the marginal cost of a unit of this system may not be adequately represented by the marginal cost of a system unit in dollars and cents since at some point the workers with the needed skills would have to be taken away from some other type of work which might also be critical to the country's well-being, or would simply not be available at all. In this example the limiting resource is the supply of workers possessing the required skills. Should the supply of properly skilled workers be the principal limiting resource, then the cost of the system should be measured in terms of the number of workers it requires. However, it seldom turns out that the supply of workers is fixed in such a manner that we can accurately measure the cost of a unit of the system under consideration in terms of skilled workers. New workers can be trained, or workers with similar skills can be retrained. Also, we seldom run into a situation where we can narrow the scarce resource down to only one item, such as the available supply of skilled workers.

In a competitive economy the monetary cost of a weapons system will furnish a fairly accurate measure of its true value in resource cost. In the case of the skilled workers, the cost of hiring the workers will eventually rise which, in turn, will cause more people to turn to the particular skills in demand. Just how well the monetary value of a weapons system will reflect its true value in resources is a complex problem, but depends generally on the extent of competition in the economy.
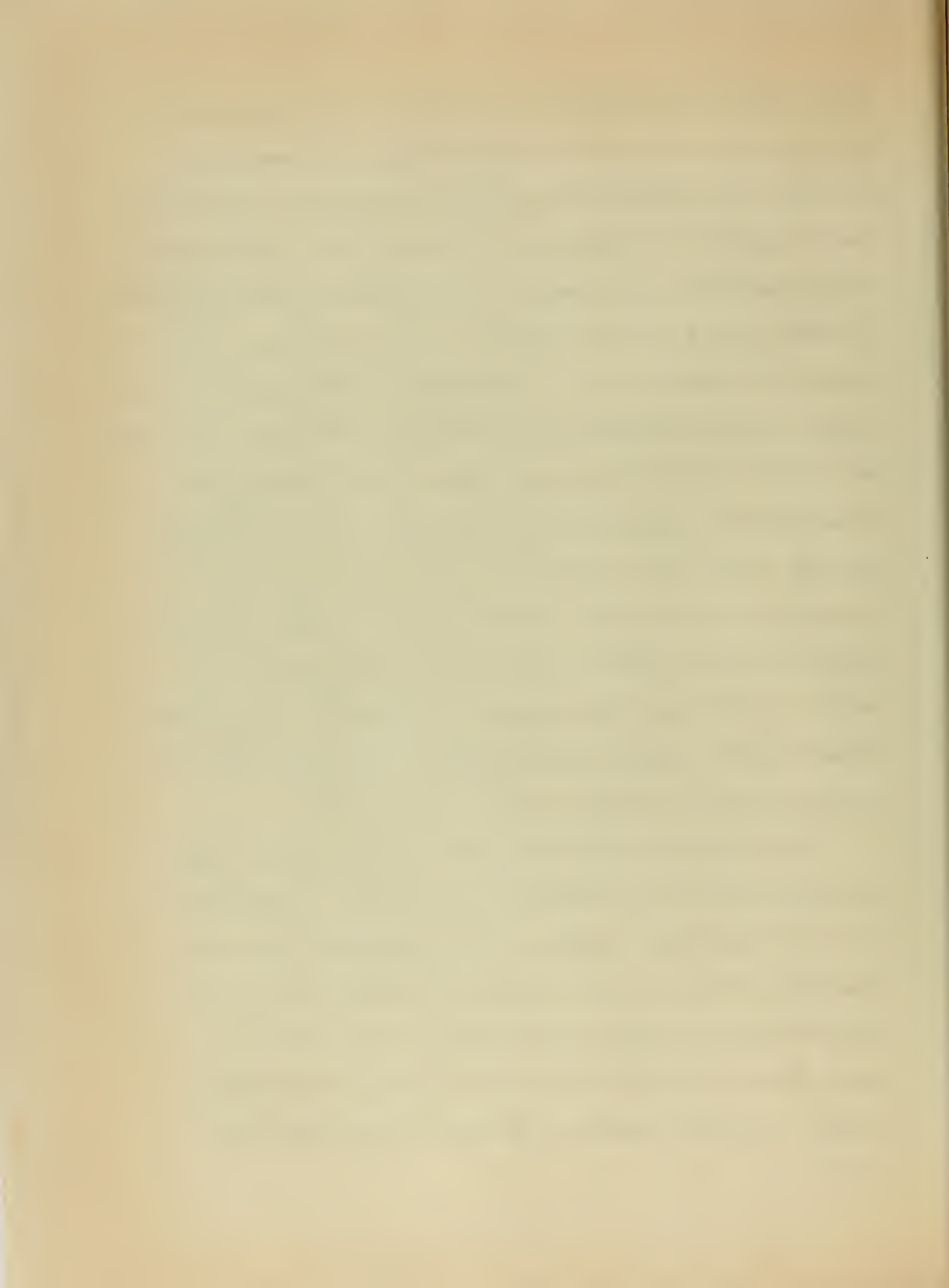
Let us now consider the specific case of the defense budget. The Defense Department faces a definite monetary constraint in the form of its annual budget. In general, the Department does not face a constraint in terms of weapons systems available, but can have more of any particular system by paying the price. In matters of advance planning the monetary constraint placed on the Defense Department is the only tangible constraint, although there are inherently many other and more complex underlying factors that constrain the country's military program. For a more comprehensive development of this topic the reader is referred to Hitch and McKean $\sqrt{3}/$.

It appears reasonable then, that defense planners should desire quantitative evaluation of various weapons systems in terms of the military effectiveness of the systems as balanced against monetary cost. This is the object of the cost effectiveness study.

4. Fixed effectiveness vs. fixed cost studies

There are two general approaches commonly used in cost effectiveness studies; the fixed cost approach, and the fixed effectiveness approach. In the fixed cost study the cost of the weapons system under consideration is initially fixed, and the system inputs are varied so as to achieve the maximum degree of effectiveness for the fixed cost. In the fixed effectiveness study, on the other hand, the degree of effectiveness to be achieved by the system is initially fixed and the system inputs are varied so as to achieve the fixed degree of effectiveness for the minimum cost.
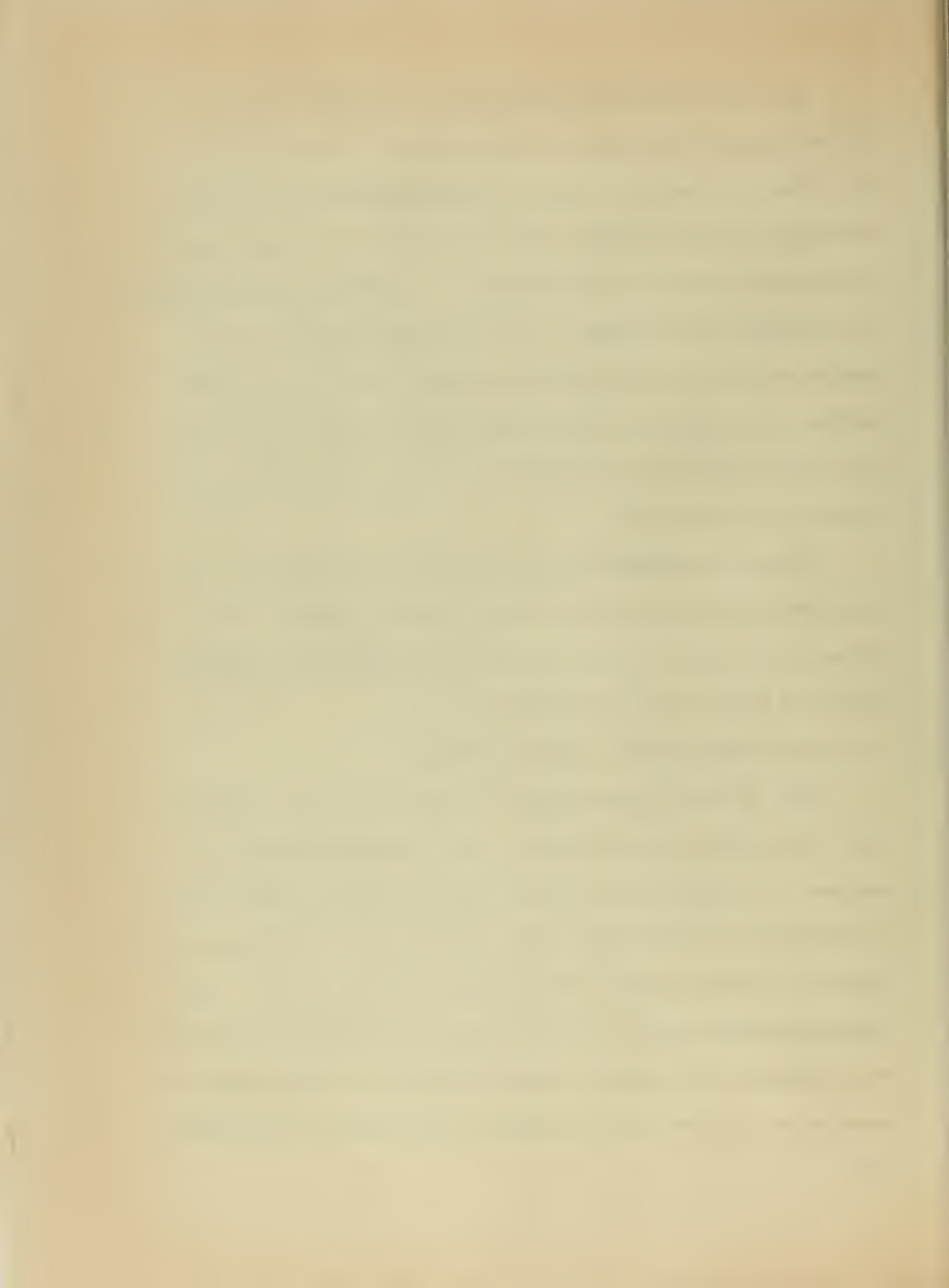
Neither of these methods provides a complete solution to the problem of selecting an optimum weapons system. The fixed cost approach does not provide a solution to the problem of selecting the initial cost, while the fixed effectiveness study does not provide an answer to the problem of how much effectiveness is needed. Thus, with each type approach, an important decision must be made either as to the amount of money that is to be put into a system, or as to the degree of effectiveness before the cost effectiveness study is undertaken. However, it is sometimes true that it is easier to arrive at an initial decision as to what degree of effectiveness is needed than it is to determine what system configurations will produce this effectiveness for the least possible cost, and vice versa. Also, several fixed effectiveness studies can be conducted to determine the cost of obtaining several different degrees of effectiveness. By conducting several fixed effectiveness studies using different degrees of effectiveness, the military planner can observe the sensitivity of cost to system effectiveness, and thus obtain information concerning how much effectiveness can be afforded. As an example, let us consider the case where a military planner is faced with the problem of determining how a certain city should be defended from air attack. It may not be difficult to decide initially that an effort should be made to defend the city, but the planner is in doubt as to the optimum mix of anti-aircraft batteries and interceptors. In this case the planner may well benefit from conducting a

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fixed effectiveness study, using as a measure of effectiveness the probability that a bomber will be able to penetrate the defenses of the city, to determine the mix between interceptors and anti-aircraft batteries that will produce a fixed degree of effectiveness for the least possible cost. Additionally, the planner might desire to look at the cost of producing several different degrees of effectiveness in order to get an idea of how much effectiveness the country could afford as balanced against the worth of the city.

Generally, the fixed effectiveness study is more applicable to weapons system selection problems than is the fixed cost study. It is usually easier to arrive at a broad decision with regard to further military needs in terms of specific capabilities, either designed to meet potential enemy threats or to support national objectives, than it is to decide initially to allot a fixed percentage of the national budget toward developing a specific military capability. Of course, this is a simplification and in reality defense planning is not sharply divided into fixed cost and fixed effectiveness thinking. Both cost and effectiveness form initial bounds within which military planning is carried out. If it turns out that the military effectiveness required to support present national objectives is so expensive as to sap the nation's economic vitality, the objectives will probably have to be revised downward, which in turn will require a lower degree of effectiveness ... etc. For purposes of this paper, it suffices to say that when analyzing weapons systems, particularly

systems which constitute a relatively small portion of the defense budget, initial decisions as to the degree of military effectiveness required is usually easier to arrive at, and is more meaningful, than is an initial decision as to how much should be spent in developing the capability.

5.   Determining the time period of a cost effectiveness study

Either implicitly or explicitly, a cost effectiveness study of a weapons system balances costs against effectiveness over some period of time. How the time period is structured into a cost effectiveness study is critical to the results obtained from the study.

If the purpose of a study is to determine preferred design characteristics of some proposed weapons system then a logical time period for the study would be the expected life of the system. Determining the life expectancy of a weapons system is a more involved problem than it first appears to be. If the life expectancy is structured simply as a fixed number of years then the study will not take into account any salvage value or transfer value the system may have at the end of the fixed time period. For instance, suppose two designs for a heavy bomber are being considered. Suppose also that one bomber design is such that at the end of the expected time to obsolescence the bomber may be converted into a tanker plane, whereas the other design does not have this versatility. If the bomber designs are considered only over the expected useful life of the aircraft as bombers, the study

obviously assigns no weight to the transfer value of the one design.
At this point the analyst must carefully consider whether the study
should compare the two airplane designs only with regard to their
suitability as bombers, or whether the study should take into account
their capability to transfer to other missions at the end of their expected
useful life as bombers. If weight should be placed on transfer capa-
bilities, how much weight should be assigned and what transfer capa-
bilities should be weighted? It is often difficult to determine the
expected life of a weapons system with respect to the designed primary
mission. This involves predicting such things as how a particular
system will adapt to subsequent alterations and how fast it will be
rendered obsolete by advances in technology. Thus it is apparent that,
although a simple time structure is often quite adequate for a cost
effectiveness study, this is a point which deserves careful thought,
particularly with respect to determining just what features of a weapons
system will be compared quantitatively by the study under the proposed
time structure.

CHAPTER II

APPLYING COST EFFECTIVENESS TO SELECTION OF PREFERRED WARSHIP CHARACTERISTICS

1. Objective

The object of this paper is to demonstrate how cost effectiveness techniques may be applied to selection of preferred warship characteristics. This paper will concern itself with the selection of certain general design characteristics of ASW surface vessels. It is emphasized, however, that the same methods are applicable to any class of surface warship.

No attempt will be made to compare the surface vessel with either aircraft or submarines designed to accomplish similar missions, although a study of the nature outlined in this paper could, with very few changes, be incorporated with similar studies conducted for aircraft and submarine systems to form a study of larger scope to determine an overall optimum mix of weapons for performing patrol and escort ASW missions.

The weapons system considered here is defined to be the ASW vessel itself. The equipment installed on board the vessel, such as guns, sonars, and radars will be considered as subsystems.

The fixed effectiveness approach will be used in developing this model. That is to say, we assume that someone has made a prior decision that a certain ASW capability is required, and that this capability is best delivered by surface, ASW vessels.

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2. Division of naval vessels into classes

Upon investigatint the various design characteristics of ships or weapons systems, and how they relate to system cost and effectiveness, it soon becomes apparent that naval vessels fall into several fairly well defined classes, or populations. For instance, when comparing speed vs. required shaft horsepower it is clear that fine-lined warships, such as cruisers and destroyers, have quite different relationships from, say, auxiliary ships such as tankers and cargo ships. However, within the bounds of any particular population the design relationships are generally of a single mathematical form.

For purposes of this study, naval vessels have been divided into five different populations; aircraft carriers, auxiliaries, minesweepers and small craft, submarines, and cruisers and destroyers. It may be noted that these divisions are generally characterized by distinct differences in hull design. As was pointed out above, it has been found, $\underline{/4/}$, that within these hydrodynamically similar populations most of the design relationships are of a single form. In this study, no attempt will be made to modify the design relationships within any population. That is, if, as in our case, one is optimizing a weapons system whose basic unit is the surface, ASW vessel, it is assumed that the variations in armament, fuel capacity, etc. will not force significant changes in the general fineness ratio and other hull design features characteristic of the destroyer-

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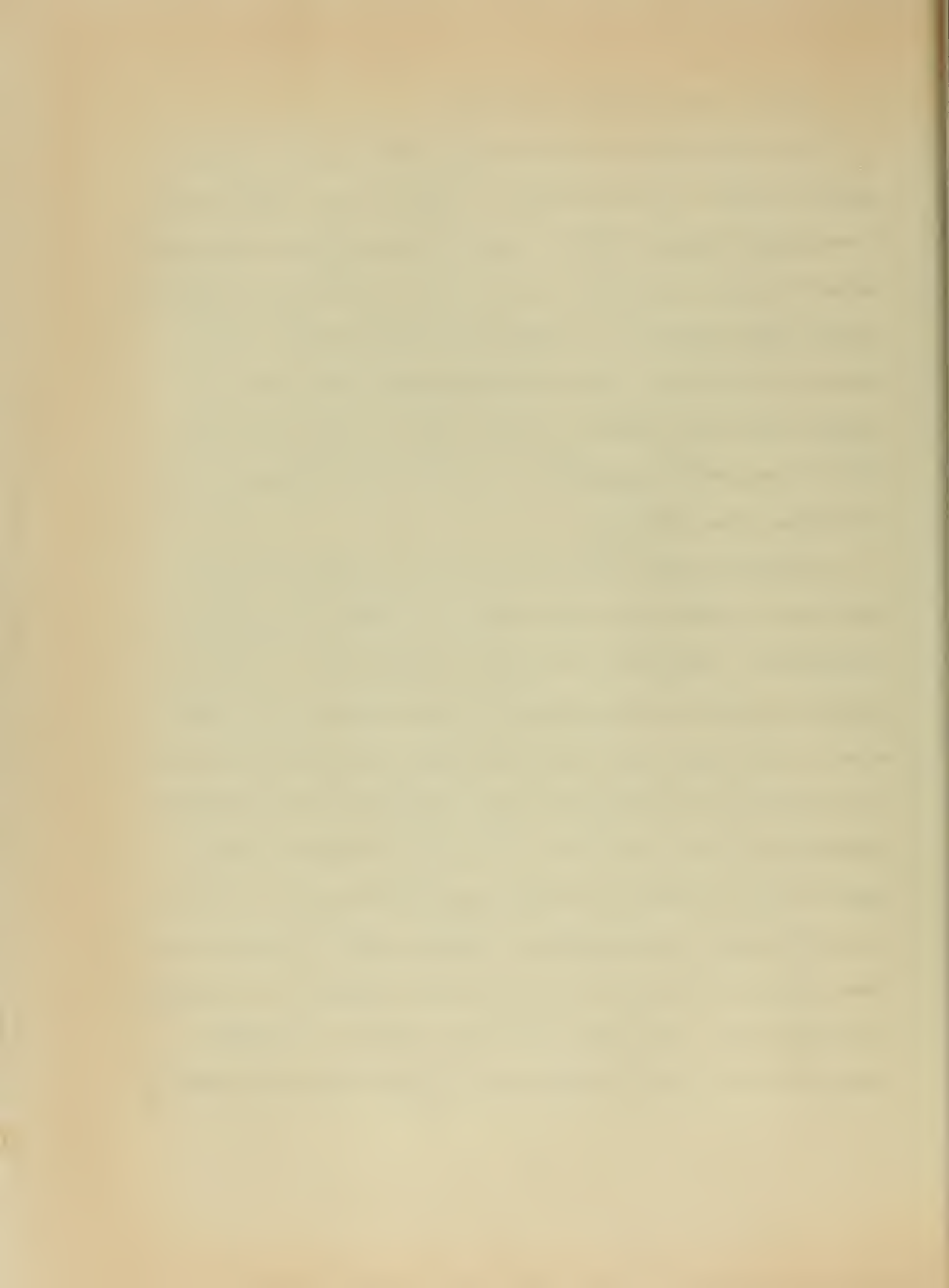
cruiser population. Considerable variation is available within each population. However, it would not alter the methodology should one wish to cross certain population boundaries by virtue of large variations in design characteristics. It would merely complicate the mathematical models used. For example, suppose it were desired to investigate the effect of enlarging the underwater sonar dome of an ASW vessel to such an extent that the vessel's speed vs. shaft horsepower relationship was changed from that of the destroyer-cruiser population to that of the mine-sweeper population which is characterized by short, broad hulls. This would simply require a provision in the mathematical model to account for the change in the speed vs. power relationships as the boundary between the populations was crossed.

Structuring the design relationships into distinct classes as above is an inflexible and rather cumbersome technique. The reason for structuring the problem in this manner is that many of the design relationships pertinent to shipbuilding are determined empirically for each new ship design. Since these relationships are determined empirically rather than analytically, quantized information on effects of departure from standard designs is difficult to come by unless one is willing to construct scale models conforming to the new design specifications and submit them to various tests in model basins and other testing apparatus.

3.   Structure of the total cost equation

The procedure used in structuring the fixed effectiveness model is to set up a total cost equation representing the cost of the weapons system under consideration. Since the fixed effectiveness approach is being considered here, the weapons system represented by the total cost equation must, of course, be capable of delivering the previously fixed degree of effectiveness. The total cost equation is first formed in terms of the major cost divisions of the system. Each cost division is then broken down to its cost elements, and each element, in turn, is reduced to a function of the significant physical characteristics of the weapons system. A cost equation structured in this manner provides a clear picture of the effect each major division, and the elements comprising the division have on total system cost. This formulation lends itself well to sensitivity analysis, and makes it possible to change certain basic assumptions which affect only one cost element without changing the entire model.

The total cost equation of the surface, ASW vessel weapons system considered in this paper consists of four major cost divisions; cost of procuring the ships required to deliver the predetermined degree of effectiveness, the cost of operating the ships over the chosen time period, the cost of procuring the support facilities required by the weapons system, and the cost of operating the support facilities over the chosen time period. These major cost divisions are thought by the author to be exhaustive and to completely represent the cost of a ship-borne weapons system.

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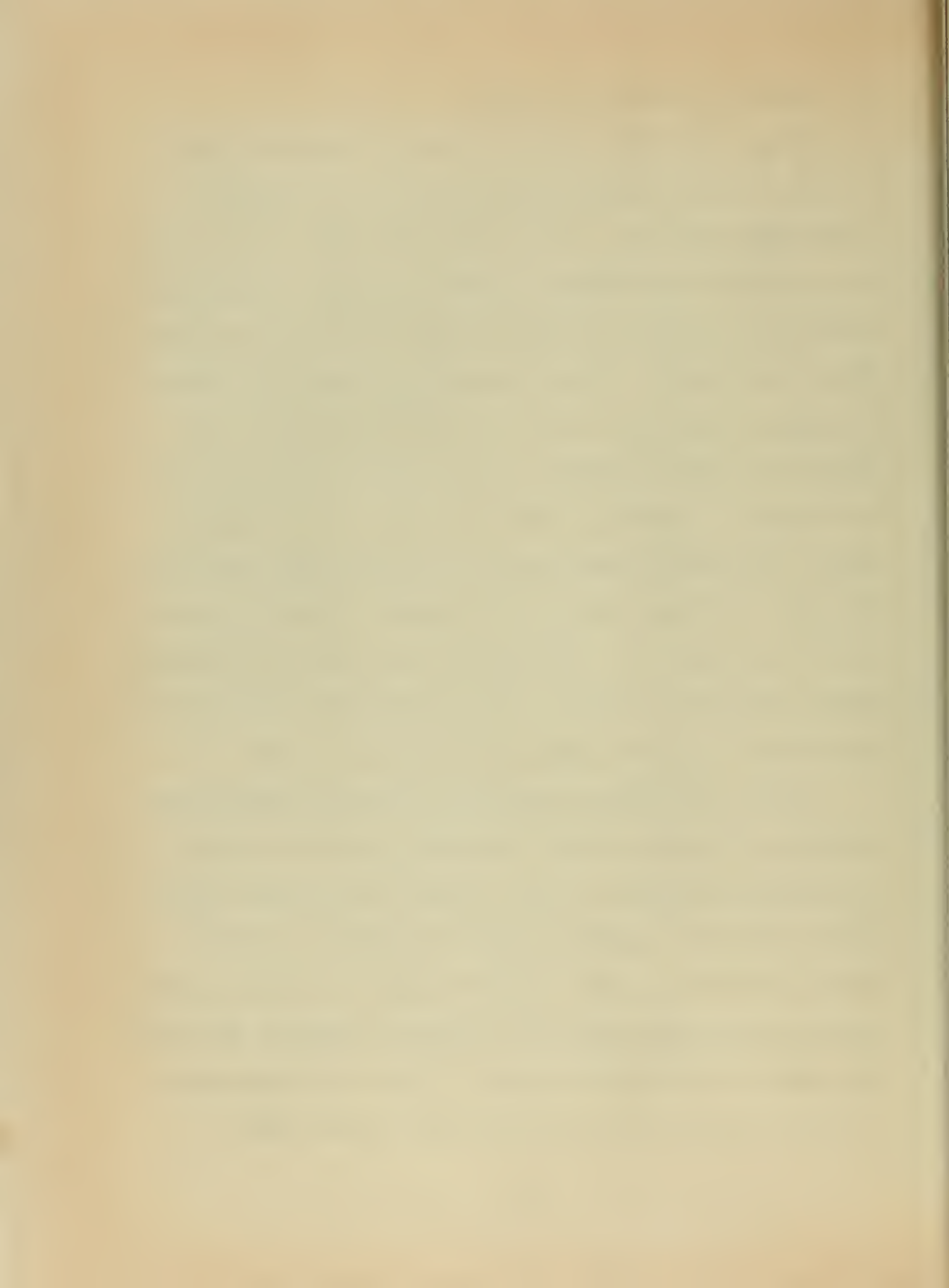
The total cost equation represented in terms of the major cost divisions is then

$$(2.1) \qquad C_T \;=\; (N)\,C_{sp} + (N)\,(Y)\,C_{os} + (N)\,C_{if} + (N)\,(Y)\,C_{of}$$

where

C_T = Total system cost (dollars) over the selected period of time

N = Number of ships required to produce the fixed level of effectiveness

Y = Time period (years) over which the system is to be considered

C_{sp} = Cost of individual ship procurement

C_{os} = Annual cost of operating one ship while performing the specified mission, or missions

C_{if} = Initial cost of support facilities required to support one ship

C_{of} = Annual operating cost of the support facilities required to support one ship.

It should be noted that N, the number of ships required to perform the assigned mission, will depend on how the measure of effectiveness used is defined, the degree of effectiveness specified, and how efficient the ships are in terms of the measure of effectiveness. Since both the measure of effectiveness used and the degree of effectiveness are initially

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specified, N must be evaluated as a function of the ship's efficiency in terms of the measure of effectiveness.

Each of the first two major cost divisions of equation (2.1) is further reduced to its component cost elements. The first two cost divisions are then represented as sums of their cost elements as

(2.2) $\qquad C_{sp} = C_h + C_p + C_{aa} + C_{asw} + C_c + C_{su} + C_{sa}$

and

(2.3) $\qquad C_{os} = C_f + C_m + C_{pe} + C_{cs}$

where

C_h $\quad = \quad$ Procurement cost of hull

C_p $\quad = \quad$ Procurement cost of propulsion

C_{aa} $\quad = \quad$ Procurement cost of anti-aircraft armament

C_{asw} $\quad = \quad$ Procurement cost of anti-submarine armament

C_c $\quad = \quad$ Procurement cost of communications equipment

C_{su} $\quad = \quad$ Procurement cost of underwater search equipment

C_{sa} $\quad = \quad$ Procurement cost of above water search equipment

C_f $\quad = \quad$ Annual cost of fuel per ship

C_m $\quad = \quad$ Annual cost of maintenance per ship

C_{cs} $\quad = \quad$ Annual cost of consumable supplies per ship

C_{pe} $\quad = \quad$ Annual cost of personnel per ship

The above equations are believed to represent the significant cost elements comprising these two major cost divisions. These cost elements are further reduced to functions of the significant physical characteristics of the weapons system in the following chapter. The order in which the cost elements are considered in the following chapter is the same as the order in which they appear in equations (2.2) and (2.3).

No attempt has been made in this paper to reduce either of the major cost divisions concerning support facilities to component cost elements. This is due principally to the unavailability of sufficient data to determine the cost of installing and operating ship support facilities as functions of the physical characteristics of the weapons system. Also, it appears to this author that the nature of the support facilities required by a ship-borne weapons system is highly tailored to fit each existing situation, and it is, therefore, not possible to specify a general mathematical form relating the cost of support facilities to the system characteristics. A heuristic discussion of the problems involved in structuring support facility costs is included below.

4.   Discussion of the problems involved in structuring initial and
      operating costs of support facilities

The principal difficulty encountered in structuring support facility costs is determining how these costs should be apportioned among the various weapons systems that make simultaneous use of the facilities.

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For example, suppose that placing into operation a new surface ship ASW system would entail the construction and operation of a certain new overseas support facility, but that this support facility would also be used by submarines and patrol aircraft. The question is, what fraction of the installation and operating costs of the new support facility should be charged to the surface ship ASW system? One solution to the problem of apportioning the initial installation costs is to try to determine if the construction of the support facility is primarily dependent on the surface ship ASW system, or if the facility would be necessary even though the surface ship system were not put into service. If it can be determined in this manner that construction of the new support facility is primarily dependent on the introduction into service of the surface ship ASW system, then the entire installation cost of the support facility, less those portions which can be directly attributed to one of the other two systems (such as the cost of constructing runways), is charged to the surface ship system. If, on the other hand, it can be determined that the existence of the new support facility is not primarily dependent on the surface ship system, then only so much of the installation cost as can be directly attributed to the surface ship system is charged to that system.

Cost effectiveness studies generally do not consider "sunk" cost when determining the cost of initial installation of support facilities. In other words, if a weapons system makes use of already existing support facilities, no attempt is made to apportion the original installation

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costs to the weapons system under consideration. For this reason the
installation cost of support facilities is not a significant cost item in
a cost effectiveness study if the weapons system under consideration
makes use of already existing support facilities.

Operating costs of support facilities are generally constant over
the range of system inputs considered in a cost effectiveness study
concerning a surface ship weapons system. If the operating costs of
support facilities are of such a nature that they are not constant over the
range of inputs considered, the relationships must be fitted to each indi-
vidual situation and no general form can be specified.

At first glance the above support facility cost procedures may seem
arbitrary and "unfair", but it must be remembered that the principal purpose
of a cost effectiveness study is to compare marginal efficiencies of the
weapons systems under consideration, not to determine accurate operating
budgets. Also, the reader might consider the problems that would arise
if an attempt were made in a cost effectiveness study to determine the
portion of sunk cost that should be assigned to each weapons system
using a support activity. A more complete treatment of this subject
may be found in Hitch and McKean $\overline{/3/}$.

CHAPTER III

STRUCTURING THE COST ELEMENTS

1. Cost of hull, C_h

If one considers ship's hulls that are generally of the same geo-
metrical shape with similar compartmentation, hull thickness, and
auxiliary fittings, it may be logically assumed that the cost of the hull
of a ship is a linear function of the displacement of the ship since a
larger displacement simply implies a larger hull requiring more labor and
material both of which have a relatively stable unit price. Since we have
restricted our investigation to populations of ships that have similarities
in these aspects, the general form of the cost of the hull can be represented
as

$$(3.1) \qquad C_h = a_1 (\Delta) + b_1$$

where Δ is the displacement of the ship in tons, and a_1 and b_1 are
constants that can be determined by statistical methods.

If the geometrical shape of the hulls under consideration are similar,
the displacement of a ship can be represented as a function of its length, $\underline{/4/}$.

$$(3.2) \qquad \Delta = (k_1) L^3 + \beta$$

From equations (3.1) and (3.2) it follows that

$$(3.3) \qquad C_h = a_h (L^3) + b_h$$

2.   Cost of propulsion, C_p

    If all the vessels considered in a study have a common type of

propulsion system, such as steam turbine drive, the cost of propulsion

machinery can be determined as a linear function of shaft horsepower.[1]

(3.4) $C_p = a_2 (SHP) + b_2$

The required maximum shaft horsepower, in turn, can be determined for

ships with geometrically similar hulls, as a function of displacement and

designed maximum speed, provided the maximum speeds of any two ships

considered are not two widely divergent, say by more than 10 knots.

Within these restrictions the maximum shaft horsepower required may be

expressed as

(3.5) $SHP = \dfrac{V^3 \Delta^{2/3}}{\copyright}$

Where V·is the maximum designed velocity of the ship and $\copyright$ is a constant.[2]

This expression also assumes that the propulsive efficiency, the ratio

between shaft horsepower and effective horsepower, is constant, an

assumption which is generally justified·within the range of accuracy

required here.

    Combining equations (3.2) and (3.4) and (3.5), we have an expression

[1] From data supplied by the Steam Turbine Division of The General
Electric Company

[2] For a discussion of the circular C and other methods of estimating
propulsion power requirements the reader is referred to /6/.

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°for the cost of propulsion as a function of maximum designed speed

and ship length

(3.6) $C_p = (a_p)V^3 L^2 + b_p$

More sophisticated expressions may be derived for required shaft

horsepower, and might be useful if it were desired to vary the geometrical

shape of the hull or the designed maximum speed over a wide range. For

a complete treatment of this subject the reader is referred to Saunders $\overline{/7/}$.

In general, however, the additional mathematical complexity encountered

when using more exact formulations for required shaft horsepower outweighs

the increases in accuracy and flexibility obtained.

3.   Cost of armament, C_{aa} and C_{asw}

In this study the cost of any particular weapon with its associated

fire control systems will be considered to be a fixed parameter. In other

words, we are assuming that the installed weapons have been subjected

to suboptimization before delivery to the ship. This is not an unreasonable

assumption since a great deal of time and effort is normally spent on

individual weapon optimization, and weapons are often developed first

with the ships being built to fit around the latest weapon design.

Consider the fixed unit price of an installed ASW weapon to be P_1

and the fixed unit price of an anti-aircraft weapon to be P_2. Cost of

installed armament can then be represented as

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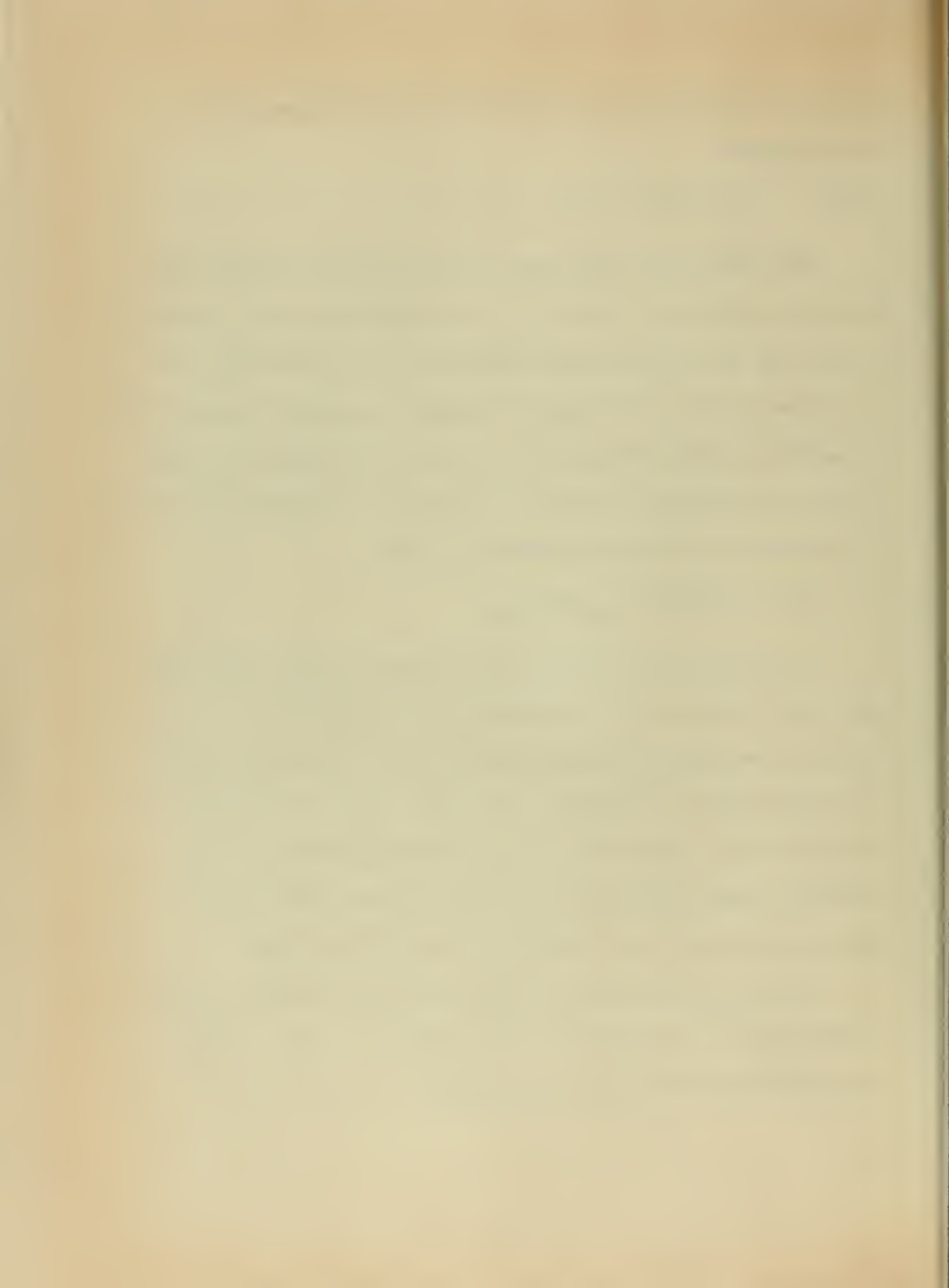
(3.7) $C_a = N_{asw}(P_1) + N_{aa}(P_2)$

N_{asw} = The number of ASW weapons installed

N_{aa} = The number of AA weapons installed

If the weight of a unit weapon is known, then it is a simple matter to re-late the displacement of the weapons installed to the size of the vessel. If we represent the displacement of one ASW unit as Δ_1 and one anti-aircraft unit as Δ_2, then we have

$$\Delta_a = N_{asw}(\Delta_1) + N_{aa}(\Delta_2)$$

If the weights of the weapons installed are known, the displacement of all the weapon units can be expressed in terms of a common ratio. In the case of two different types of units as illustrated above this relation becomes

$$\Delta_2 = k_2(\Delta_1)$$

and the expression for the displacement of armament becomes

(3.8) $\Delta_a = \Delta_1 (N_{asw} + k_a N_{aa})$

Then from equation (3.2) it follows that a general relationship between the amount of installed armament and the ship's dimensions may be obtained. Now, if we wish to consider the effect of, say, holding the number of installed anti-aircraft units constant and varying the number of ASW installations, we have

(3.9) $\Delta_a = \Delta_1 N_{asw} + A'$
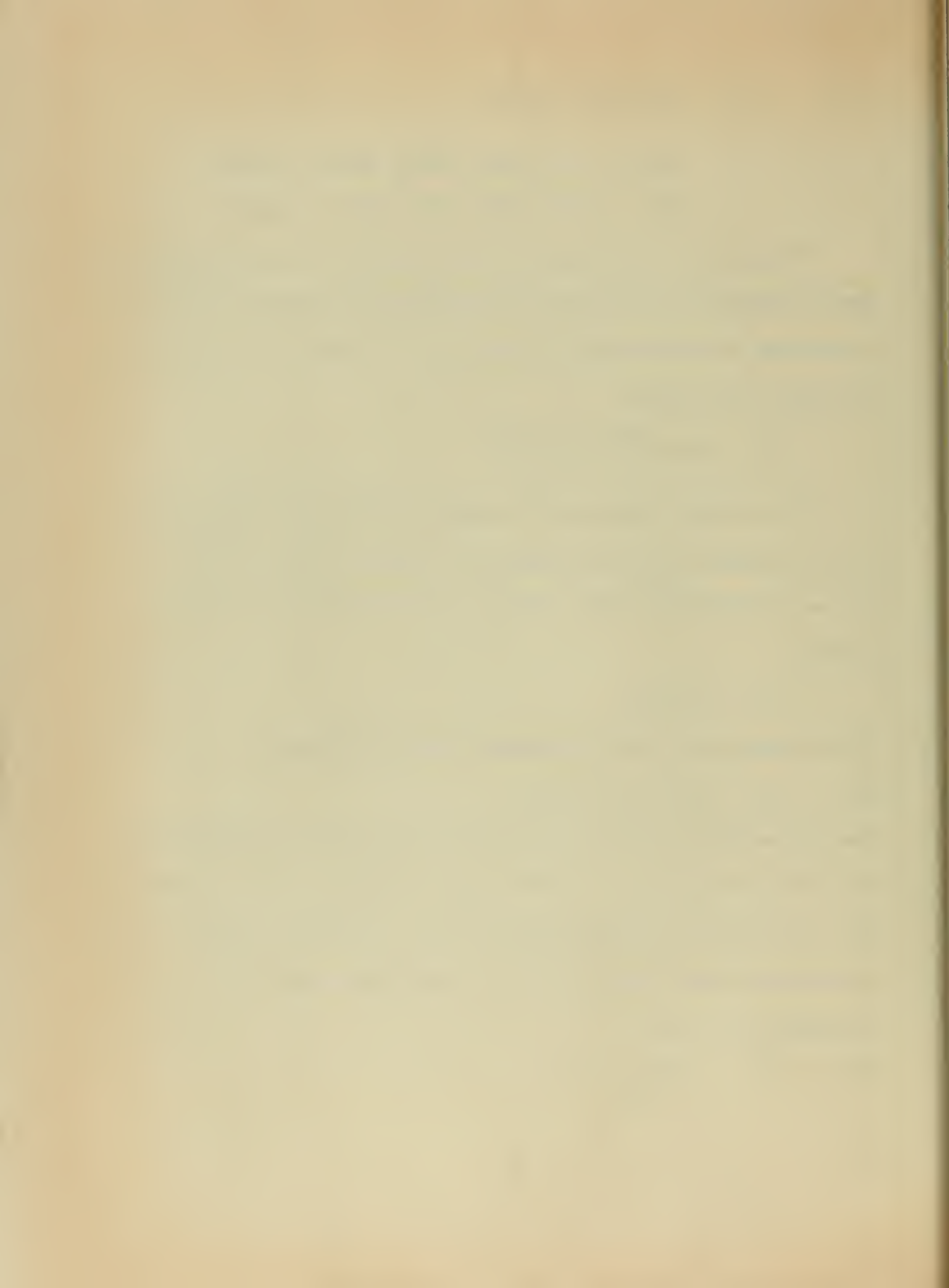
Thus it is not only possible to determine the direct costs of increasing

or decreasing the quantity of installed armament, but also how these

changes affect other interconnected ship characteristics.

It should be pointed out that varying the amounts of installed armament

will very likely influence the ships performance in terms of its measure of

effectiveness. In the case of the fixed effectiveness study a change in

ship performance will simply be reflected by either increasing or decreasing

the number of ships required to produce the predetermined degree of effec-

tiveness.

4. Cost of communications equipment, C_c

No attempt has been made in this paper to determine the cost of

shipboard communications equipment as a function of system characteristics.

There is very little data available in this field. Also, it is difficult to

determine what physical characteristics of the weapons system one should

relate to the cost of communications equipment. It appears to the author,

however, that in most circumstances the cost of communications equipment

installed on board a population of naval vessels can be considered as a

constant. It is true that there are certain cases where significant changes

in the cost of ship procurement and the military capability of a ship are

brought about by changes in the installed communications equipment,

such as in the case of installation of the Naval Tactical Data System,

but for the most part it appears that within a given population of ships,
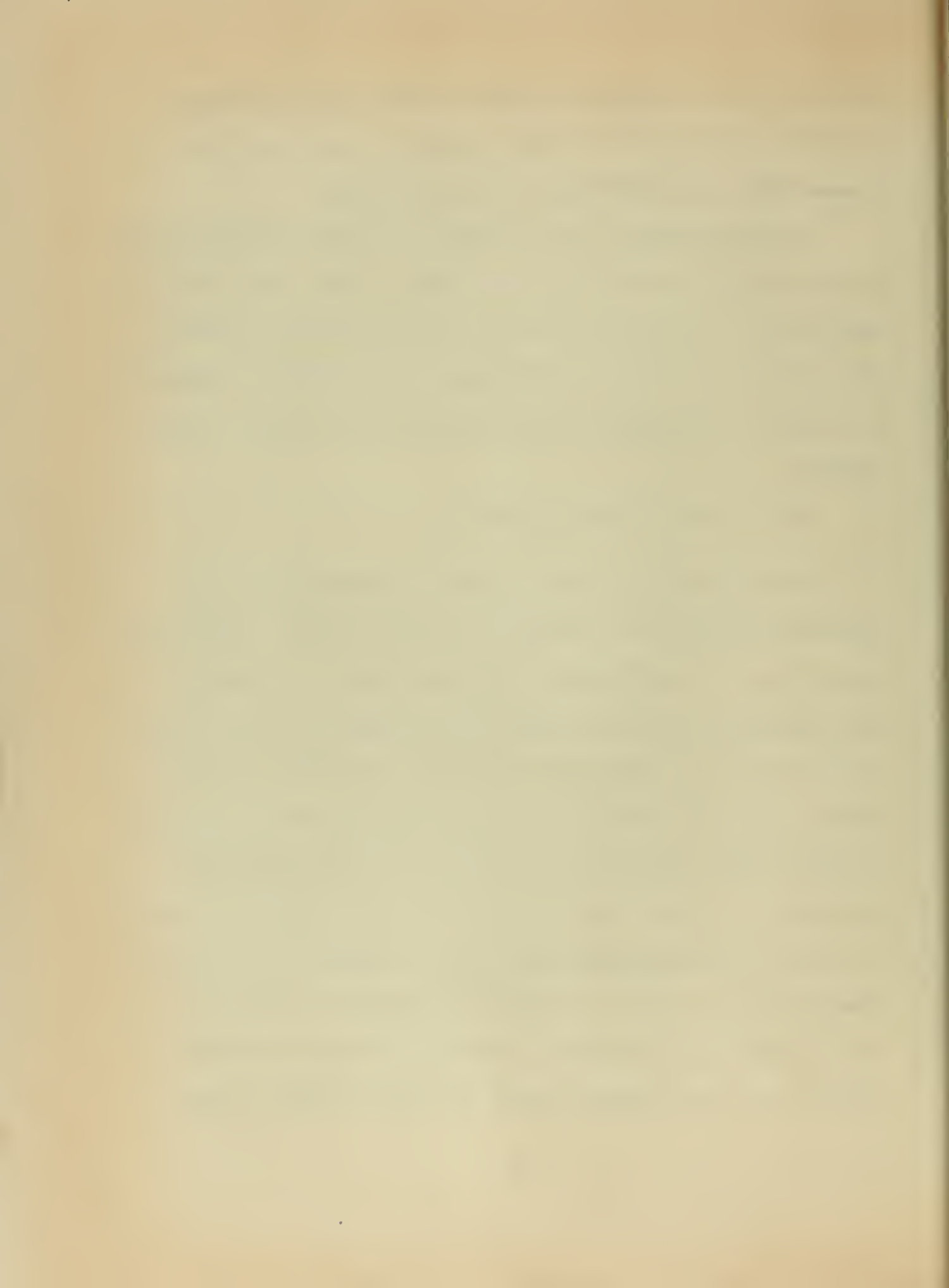
there are a few significant changes in ship cost or performance due to changes in the installed communications equipment.

5.   Cost of search equipment, C_{su}, and C_{sa}

The principal cost factor for both underwater and above water search equipment is the cost of installed electronic equipment. Only the cost relationships for underwater search equipment are analyzed in this paper. However, it seems reasonable to assume that relationships between effectiveness and cost of above water electronic search equipment similar to those derived in appendix A for sonar equipment exist, and could be formulated if desired. Since the vessel under consideration here is primarily an ASW weapon, its costs and effectiveness are sensitive to variance of sonar parameters. For many studies of ASW weapons systems it may be acceptable to fix the cost of installed above water search equipment at a constant representative figure.

The underwater search equipment considered in this study is the echo ranging sonar of a general type being currently installed in destroyers and destroyer escorts of the U. S. Navy. There has been little interest shown to date by the U. S. Navy in passive sonars for use in surface vessels, therefore only active sonar is considered.

It can be shown[1] that sonar detection ranges vary inversely with the surface area of the underwater transducer. This is an important relationship

[1]See Appendix A

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because after a relatively constant initial cost of installing the electronic circuitry and other components of a typical echo ranging sonar, the cost of the sonar installation varies directly with the surface area of its transducer. The explanation for this general linear relationship between cost and transducer surface area lies in the increased amounts of expensive materials used in constructing the sensitive elements of a transducer, and in the fact that transducer sizes are increased to accommodate larger and more expensive power generating equipment. The cost of underwater search equipment can then be represented as a function of detection range

$$(3.10) \qquad C_{su} = a_{su} \left(R_u \right) + b_{su}$$

where R_u is the sonar detection range, b_{su} the fixed electronic installation cost, and a_{su} a proportionality constant.

One might also be interested in determining the relationship between the increasing surface area of a sonar transducer and the resulting increase in shaft horsepower required to maintain the same maximum speed. This depends a great deal on where the sonar transducer is located along the underwater body of the ship. The present tendency is to locate the sonar transducer under the bow of a ship as much as possible as the transducer sizes become great enough to create a significant drag effect. It is theoretically possible to actually increase a ship's streamline with a bow mounted sonar provided the transducer housing is made so as to take

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advantage of the so called "bulbous bow" effect, although this has

seldom been realized in practice. However, let us suppose that the

original design of a particular ASW vessel had the sonar located slightly

forward of the amidships position, and it is desired to calculate the effect

of increasing the size of the sonar transducer without altering the other

hull dimensions. In this position the drag, and accordingly the required

shaft horsepower, varies directly with the surface area of the transducer

housing. A rough calculation for the additional shaft horsepower required

may then be made by computing the fraction of the original transducer

area to the original hull surface area, $\dfrac{h^2}{L^2}$, and letting SHP represent

the total shaft horsepower, we have

$$SHP_1 = (SHP) \cdot \frac{h^2}{L^2} \, k_3$$

$$SHP_1^0 = (SHP) \frac{h'^2}{L^2} \, k_3$$

(3.11) $SHP' = SHP - SHP_1 + SHP_1^0$

where h' represents the new linear dimension of the sonar transducer,

and SHP' the new total shaft horsepower required.

6.   Cost of fuel, $\underline{C_f}$

The amount of fuel consumed by a ship is directly proportional

to the propulsion power utilized. If all ships considered have the same

type of propulsion machinery; e.g., all steam turbine drive, then the
amount of fuel used by a ship is a function of its operating speed, or, more
specifically, a function of some weighted average of speeds employed by
the ship during its operating period. Since propulsion power utilized
varies exponentially with the speed of a ship, a time weighted average
of the ship's operating speeds is required to determine fuel consumption.
However, for purposes of comparing two ships with identical missions and
comparable maximum speeds, it is reasonable to assume that each ship
will spend approximately the same fraction of total mission time at speeds
other than the required cruising speed. That is to say, that if we were
comparing the fuel consumption of two ships on a patrol mission, we
calculate each ship's fuel consumption at the average speed required to
accomplish the patrol mission in the allotted time, and that each ship
will spend a comparable length of time at speeds above and below this
required cruising speed.

The cost of fuel can then be expressed as a linear function of the
propulsion power necessary to maintain the required cruising speed.

(3.12) $\qquad C_f = a(SHP_c)$

The expression for cruising horsepower as a function of cruising speed,
v_e, is

(3.13) $\qquad SHP_c = k_4 (SHP) \left(\frac{v_e}{V} \right)^g$

where k_4 and g are curve fitting constants obtained by fitting power vs.
speed curves for vessels of the same general hull design. This expo-
nential representation of cruising horsepower will provide accurate
results over a speed range of from about eight knots to 95% maximum
speed. At speeds below eight knots the shaft horsepower vs. speed
relationship is generally linear. For most purposes warship cruising
speeds are in excess of eight knots, and equation (3.13) is an adequate
representation of cruising propulsion power. It may seem strange, at
first glance, that equation (3.5) is not used directly to compute cruising
shaft horsepower requirements, but it must be remembered that equation
(3.5) itself is an empirical equation which has been found to hold as a
result of past ship design experience. Equation (3.5) is specifically
formulated for computation of maximum shaft horsepower requirements,
the precise region in which equation (3.13) gives inaccurate results.
In order to use equation (3.5) directly for computation of cruising
horsepower requirements, it would be necessary to use different values
of the constant Ⓒ, to fit the various speed ranges. Equation (3.13)
actually does just that in a slightly different form.[1] From equations
(3.12) and (3.13) it follows that

(3.14) $C_f = a_f \, (SHP) \, (\frac{V_e}{V})^g \; + b_f$

[1]Some examples of speed vs. power, and power vs. fuel con-
sumption curves are provided by Hauschildt; M. V. Hauschildt, Con-
siderations Affecting the Design Endurance of Naval Ships, Transactions
of The Society of Naval Architects and Marine Engineers, volume 65, 1957,
and by Pien and Todd; P. C. Pien and F. H. Todd, The Effect Upon Resistance
and Power of Variation of LCB Position, Transactions of The Society of Naval
Architects and Marine Engineers, Volume 64, 1956.
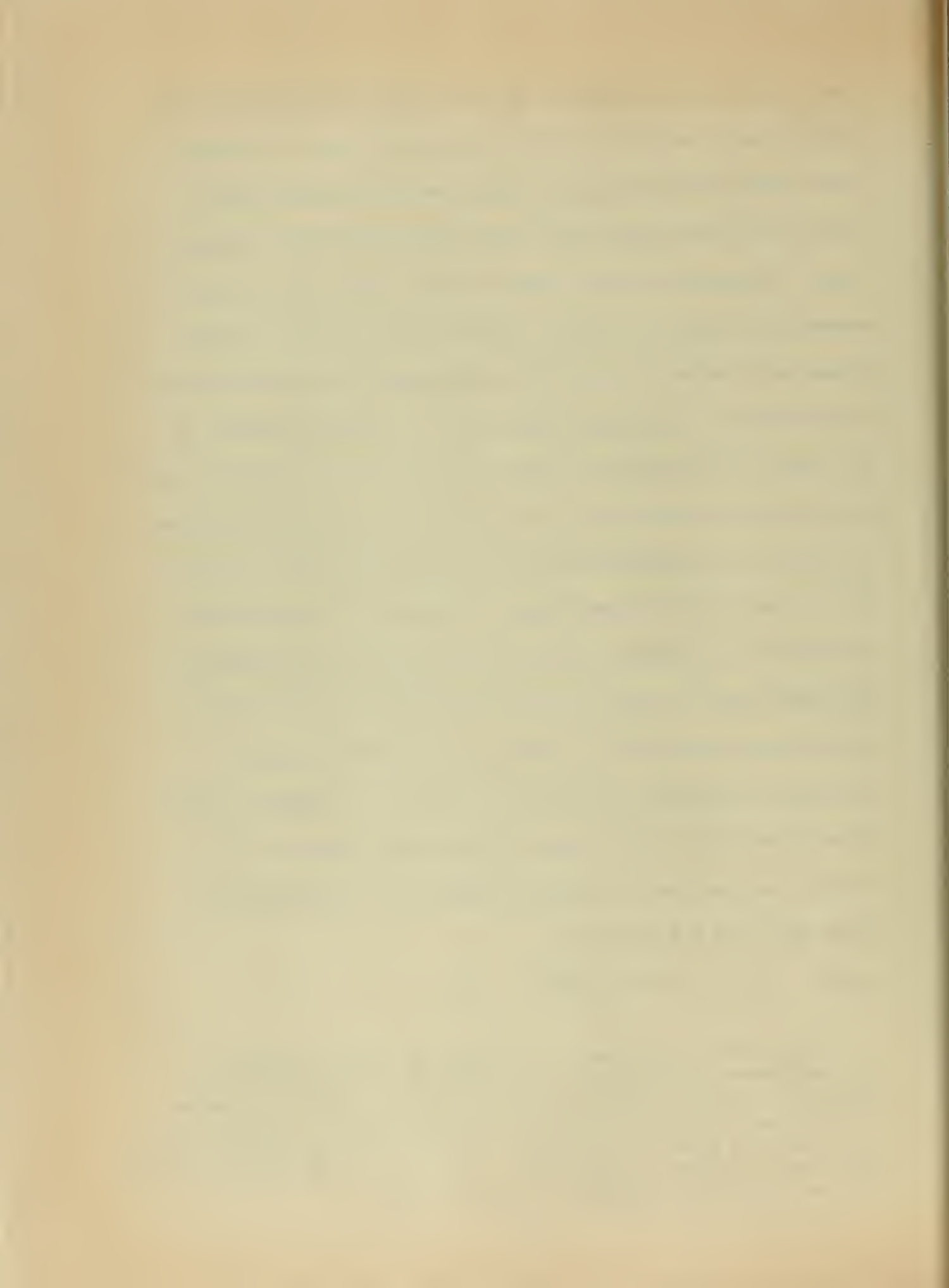
7.   Cost of maintenance, C_m

Maintenance costs are divided into two major components; cost of regularly scheduled overhauls, and cost of necessary maintenance other than that accomplished at regularly scheduled overhauls, referred to hereafter as "restricted availability". To a first order of approximation, the annual cost of scheduled overhauls can be considered to be a fixed fraction of the total procurement cost of a ship, $\underline{/5/}$. Unfortunately, the present system of accounting for restricted availability funds does not lend itself well to an analysis of the type required for a cost effectiveness study. That is, the restricted availability costs are not related to the various system operating variables. It seems reasonable to assume that restricted availability costs vary principally as a function of cruising speed. At cruising speeds of below 15 knots restricted availability costs are probably relatively insensitive to speed and could be estimated as a fixed fraction of scheduled overhaul costs. At cruising speeds over 15 knots it seems reasonable to postulate that additional restricted availability costs are of the form

(3.15) $C = a_{m1}(v_e - 15) + a_{m2}(v_e - 15)^2$

$a_{m1} > a_{m2} > 0$.

Structuring the coefficients a_{m1} and a_{m2} as constants introduces a bias in favor of large, expensive ships. This is done on the assumption that

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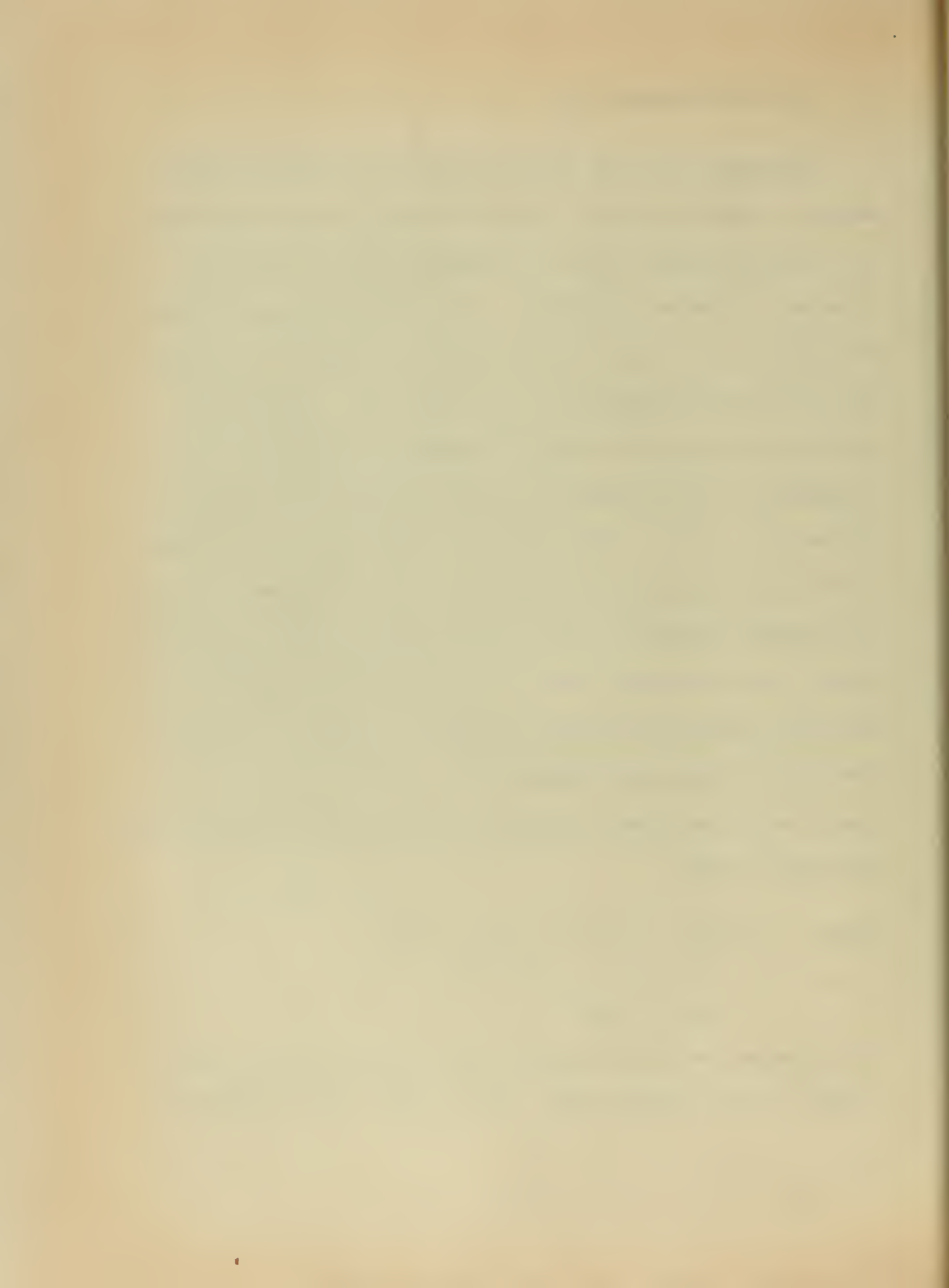
the relative increase in maintenance costs for operating small ships
at high speeds will be greater than that for large ships. As a result
of the above assumptions, the annual cost of maintenance can be
expressed as

$$(3.16) \qquad C_m = \begin{cases} k_m (C_{sp}) & \text{if } v_e \leqq 15 \\[2em] k_m (C_{sp}) + a_{m1} (v_e - 15) + a m_2 (v_e - 15)^2 \\[1em] \qquad\qquad\qquad \text{if } v_e > 15 \end{cases}$$

8.   Cost of personnel, C_{pe}

As in the case of non scheduled maintenance costs, the Navy's
personnel costing procedures are not structured in such a manner as to
be readily adaptable to a cost effectiveness study. Here again, however,
putting the existing information into a format usable for cost effectiveness
studies would not be conceptually a very difficult task, even though it
might prove arduous in execution. It is relatively easy to arrive at a
representative cost figure for an officer or enlisted man of any particular
rank in terms of salary, allowances, basic training supplied by the Navy,
etc. The major difficulty lies in determining the requirements of a par-
ticular weapons system in terms of the number and rank structure of the
personnel required to man the system, the amount and cost of the special-
ized training required over and above the average basic training, and
how these costs are related to the system operating variables.

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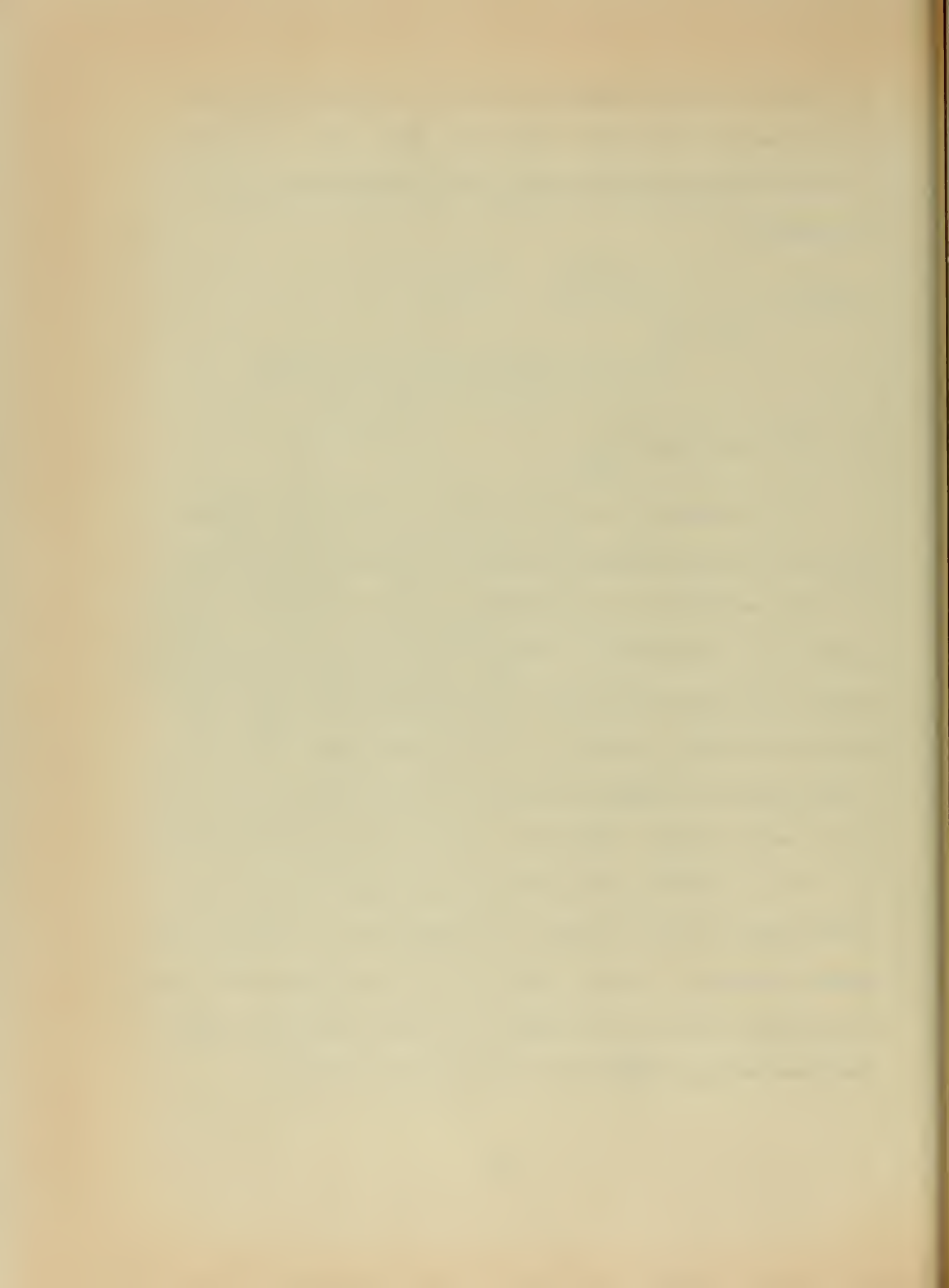
The operating costs attributed to personnel required to operate the ship's propulsion and associated auxiliary machinery varies linearly with shaft horsepower, whereas costs attributed to general shipkeeping personnel vary linearly with the displacement of the vessel. Using these relationships and the already established relationships between shaft horsepower, displacement, and length, this particular segment of personnel cost takes the form

(3.17) $\quad C = a_{pe1}(V^3L^2) + a_{pe2}(L^3) + b_{pe}$

In the absence of supporting data, it is postulated that the operating cost due to personnel required to man the electronic search equipment, both above water and under water, is a linear function of the detection range of the equipment.

(3.18) $\quad C = d_{pe1} + d_{pe2}(R)$

The operating cost due to personnel required to man the installed communications equipment is considered a constant in keeping with earlier assumptions. Since the installed armament is considered to be a parameterized value, the technically trained personnel required for the maintenance of the ordnance equipment is also considered a parameterized cost.

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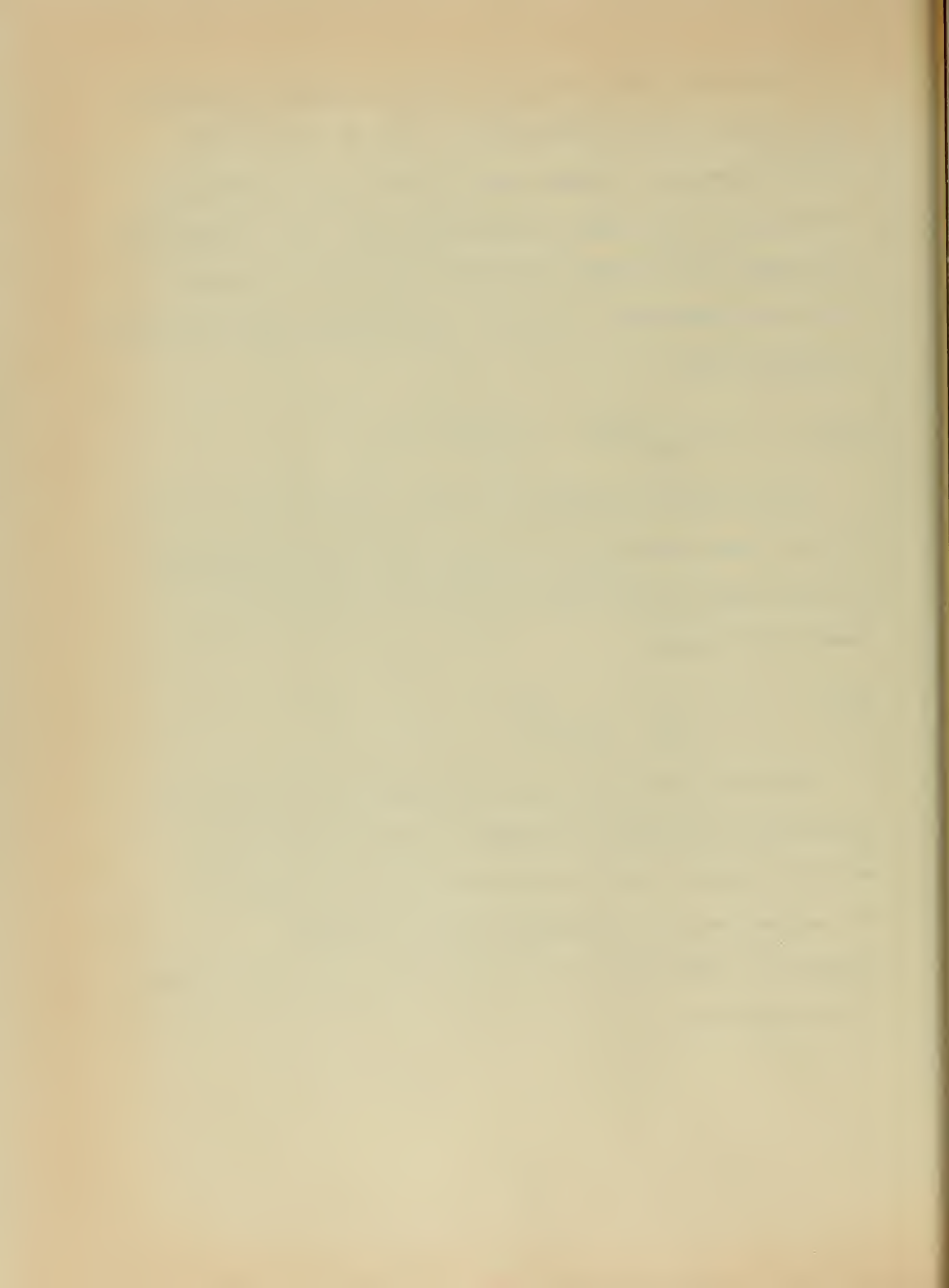
9.   Cost of consumable supplies, C_{cs}

Standard Navy publications are available in the NWP series which serve as planning guides for provision usage rates and equipage rates. Provision usage rates per man per day along with representative costs may be obtained from these publications. Cost of equipage is considered a constant here since any significant variance in usage rates will be covered under restricted availability. Cost of consumable supplies is not generally a significant item for purposes of cost effectiveness comparisons, although, as will be illustrated later, the storage space used by consumable supplies is a significant factor in determining endurance and ship sizes.

CHAPTER IV

ASSEMBLING THE COST ELEMENTS

1.   The mission

The mission, or missions, to be performed by a weapons system
under consideration in a cost effectiveness study must be specified by
the military planners who are to make use ot the study. Once the mission
is specified, it must be represented mathematically in such a manner as
to determine the number of weapons system units, in this case the number
of ships, required as a function of the effectiveness of the system in per-
forming its assigned mission. It is important that the mathematical struc-
ture of the mission be such that the effect of varying the weapons system
design characteristics on the number of system units required is readily
apparent.

Obviously the mathematical structure of the mission must be deter-
mined individually to fit each particular case. Thus there is no general
algorithm available for structuring a mission mathematically. The following
two examples are intended to illustrate the general approach to be used in
structuring the mission of the surface ship, ASW weapons system considered
in this paper.

Example A; Patrol Mission

Consider a very simple formulation of an ASW patrol mission of the
type a surface ship, ASW weapons system might be called upon to perform

during a cold war situation, or on the periphery of a limited war such as the Korean conflict. It is desired to cover a specified area, A, by sonar surveillance within a specified period of time on a continuing basis. The time period can then be expressed as the revisit rate at any point in area A. The definite range of detection law is to be used for computing sonar sweep widths.[1]

Under the above assumptions the number of ships required in area A at all times to accomplish the assigned task is

$$(4.1) \qquad N_1 = \frac{(A) \ (1000)}{R_u v_e \ 24 \ T}$$

where A is the specified area to be covered (sq. miles), R the 50% probability sonar detection range (yards), T the revisit time (days), and v_e the cruising speed of the patrol vessel (knots). The number of ships that must be contained in the system to maintain one ship on station continuously is

$$(4.2) \qquad N_2 = \frac{E + R_1}{E - \frac{2 \ d}{24 \ v_e}}$$

where E is the endurance of the patrol vessel (days), R_1 the number of days required for replenishment and repair between patrols, and d is the average distance from the support base to the patrol area (miles). R_1 can usually be adequately expressed as a fraction of E.

[1]See Appendix B for derivation of definite range law of detection

Combining equations (4.1) and (4.2) the number of ships, N, in the total cost equation is

$$
(4.3) \qquad N = \left[\frac{(A) \; 1000}{R \; v_e \quad 24 \quad T} \right] \left[\frac{E + \dfrac{R_1}{}}{E - \dfrac{2d}{24 \; v_e}} \right]
$$

Example B; Convoy Escort Mission

The mission in this example is to provide complete sonar coverage across the front of a convoy[1] of width W (yards) proceeding at convoy speed, v_c, using the definite range law of detection. In this case it may be shown[2] that the number of ships in the convoy escort must be at least

$$
(4.4) \qquad N_1 = \frac{W \, v_c}{R \sqrt{v_e^2 + 3 v_c^2}} \qquad \left\{ v_e \geq v_c \right\}
$$

If it is assumed that there are no support facilities between the initial and terminal points of the convoy, and no provisions for replenishment at sea, then the endurance of the escort vessel must be at least equal to the time required for a convoy crossing. Hence, if the length of the convoy route is d (miles), the time required for one transit is

[1] This concept of a straight line screen across the front of a convoy can be easily expanded to the more common types of ASW screens; circular, semi-circular, or horseshoe.

[2] See Appendix C.

$\dfrac{d}{24\ v_c}$, and $E \geq \dfrac{d}{24\ v_c}$.   Then, if continuous operation of convoys is assumed, the total number of ships required to maintain one ship on continuous escort duty is

$$(4.5) \qquad N_2 = \dfrac{\dfrac{d}{24\ v_c} + R_1}{\dfrac{d}{24\ v_c}} \ .$$

Combining equations (4.4) and (4.5) the number of ships required, (N), for the total cost equation is

$$(4.6) \qquad N = \left[\dfrac{W\ v_c}{RV\,v_e^2 + 3v_c^2} \right] \left[\dfrac{d + (24)\ (v_c)\ (R_1)}{d} \right]$$

It is worth while to pause here and observe some of the properties of this solution for N. First, it is obvious that if the cruising speed of the escort is less than that of the convoy, the escort will not be capable of performing its task. Also, it may be observed from (4.4) that if $R = \dfrac{W}{2}$, then $N_1 \leq 1$, which may be interpreted to mean that only one escort is needed for the convoy, and that the escort speed need be no greater than the convoy speed. If $v_e = v_c$, then $N_1 = \dfrac{W}{2R}$. These results are intuitively obvious. It might also be of interest to look at the relationship between escort speed and the number of escort vessels required to provide coverage for a given size convoy traveling at a specified speed over a specified route. For this purpose equation (4.6) can be put into the form of a hyperbola, fig. 1,
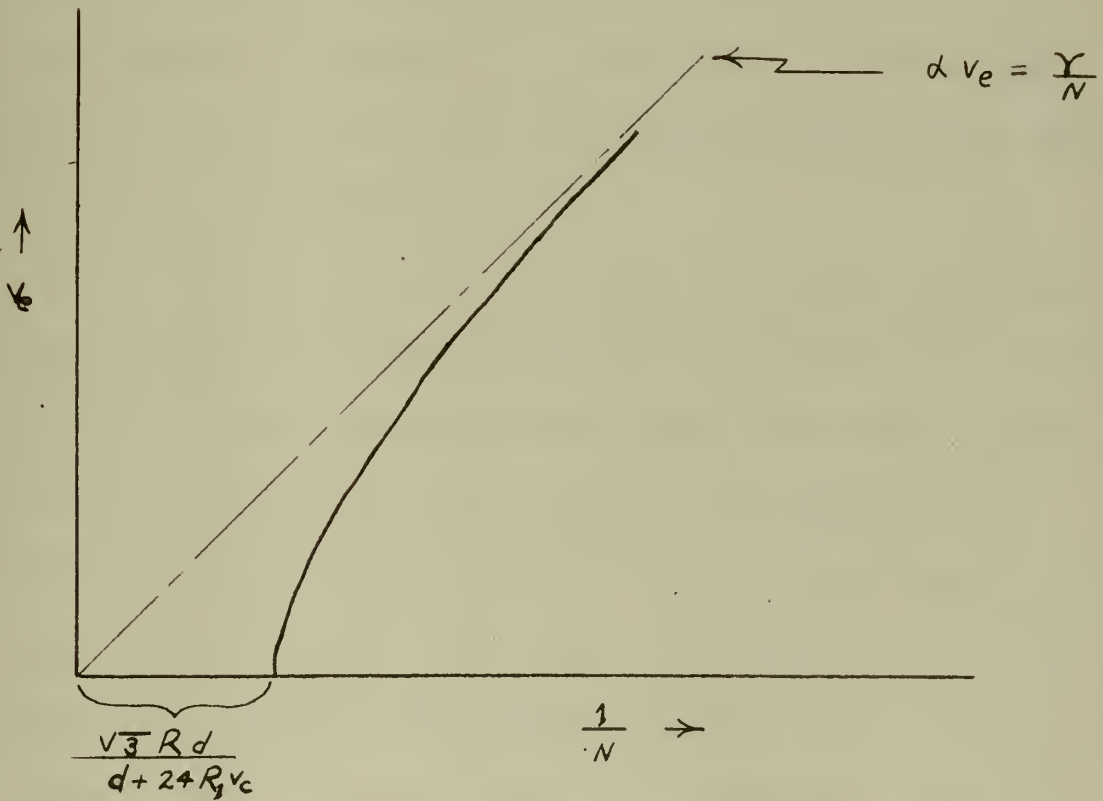
(4 1)

(4.7) $\dfrac{\left(\frac{1}{N^2}\right)}{\alpha^2} - \dfrac{v_e^2}{\gamma^2} = 1$

where

$$\alpha = \frac{\sqrt{3}\,R\,d}{d + 24\,R_j\,v_c}$$

$$\gamma = \sqrt{3}\,v_c$$

$\alpha\,v_e = \dfrac{\gamma}{N}$

v_e

$\dfrac{\sqrt{3}\,R\,d}{d + 24\,R_j\,v_c}$

$\dfrac{1}{N}$

Number of Escorts Required vs. Escort Speed

Figure 1

From figure 1 it is apparent that N asymptotically approaches the form

(4.8)

$$\left[N = \left[\left(\frac{d + 24v_c R_1)\ v_c}{Rd} \right) \right] \left[\frac{1}{v_e} \right] \right]$$

An equation such as (4.8) is useful for sensitivity of input analysis.
For instance, from equation (4.8) it can be seen that the number of ships
necessary to meet the specified escort requirements is very sensitive to
convoy speed since N is dependent on the second power of the convoy
speed. While this fact may not be of significance to this particular
study, it suggests to the analyst that a study of optimum convoy speeds
is likely to be a fruitful endeavor.

Let us suppose that it has been determined that it is feasible to
place either one or two ASW batteries aboard the convoy escort vessel
and it is desired to investigate the trade offs between the kill proba-
bilities of the one and two weapon ships and their costs. Let us
additionally suppose that by experimentation and other studies it has
been determined that the kill probability, given detection, of the one
weapon ship against a single submarine is 0.30, whereas the kill
probability of the two weapon ship is 0.55. Assume also that it has
been determined that when more than one ship attacks a submarine the
probability of kill is as if the ships were attacking independently.

Using the above assumptions we see that if all the ships in a
convoy screen join in an attack on a submarine penetrating the screen,

the kill probability employing single weapon ships is

$$P = 1 - (1 - .30)^N$$

whereas the kill probability of the two weapon ships against a single penetrator is

$$P = 1 - (1 - .55)^N$$

Thus, as the number of ships participating in an attack increases, the advantage of the two weapon ships rapidly decreases. Of course, the tactic of having all ships in a screen participate in an attack may not be the most desirable, but this simple example illustrates how the structure of the mission in a cost effectiveness study may be used to gain several simultaneous bits of information.

2.   The total cost equation

After the mission has been defined, it remains to put all the various cost elements together in a meaningful fashion. Here again, the exact manner in which the cost equation is assembled depends on the information desired from the model and must be fitted to suit each situation.

As an example, let us consider a weapons system consisting of surface ASW vessels intended to perform patrol mission A as outlined above. Using the cost elements developed in chapter II, a typical formulation of the total cost equation is illustrated below.

$$(4.9) \quad C_T = \left[\frac{A(1000)}{R_u \, v_e \, T \, 24} \right] \left[\frac{E + R_s}{E - \frac{2d}{24 \, v_e}} \right] \left\{ a_h (L^3) + b_h + a_p \, V^3 L^2 + b_p \right.$$

$$\left. + N_{asw} (P_1) + N_{aa} (P_2) + C_c + q_{su}(R_u) + q_{sa}(R_a) + b_s \right\}$$

$$+ \left[\frac{A(1000)}{R_u \, v_e \, 24 \, T} \right] \left[\frac{E + R_s}{E - \frac{2d}{24 \, v_e}} \right] \left[Y \right] \left\{ q_f \, V^3 L^2 \left[\frac{E}{E + R_s} \right] (365) \left(\frac{v_e}{V} \right)^3 \right.$$

$$+ b_f + \left[\begin{array}{l} k_m (C_{sp}) \quad \text{if} \quad v_e \le 15 \\[2mm] k_m (C_{sp}) + a_{m1} (v_e - 15) + a_{m2} (v_e - 15)^2 \text{ if } v_e > 15 \end{array} \right]$$

$$+ q_{pe_1} (V^3 L^2) + q_{pe_2} (L^3) + q_{pe_3} (R_u) + q_{pe_4} (R_a)$$

$$\left. + b_{pe} + C_{cs} \right\} + NY C_{of}$$

where the endurance of the ASW vessel (E in equation 4.9) may be

determined from a knowledge of the rates of fuel consumption and

consumable supplies consumption.

(4.10) $\quad k_1 L^3 + \beta = \delta_p(V^3 L^2) + \delta_{P_3} + E \delta_f\left[k_4(V^3 L^2)\left(\frac{V_e}{V}\right)^a\right]$

$+ N_{asw}(\Delta_3) + N_{aa}(\Delta_2) + E\left[\delta_{cs} k_1 L^3 + \delta_{cs_3}\right] + \delta_{su} R_u^{3/2} + \delta_{sa} R_a^{3/2} + \delta_s$

$+ \Delta_c + k_h L^3 + \delta_{pe}\left[k_1 L^3 + \beta\right] + \Delta_k$

It may be noted from equation (4.10) that the number of personnel in the
crew is considered to be a linear function of displacement of a ship. For
endurance computations this is an adequate estimate. The term
of equation (4.10) is simply an additional displacement not accounted
for by the other listed displacement divisions. For most classes of ships
the first eight terms to the right of the equality sign in equation (4.10)
will be an adequate representation of displacement, in which case

Solving equation (4.10) for E we have

(4.11) $\quad E = \left(k_1 L^3 + \beta\right) - \left\{ \delta_p(V^3 L^2) + \delta_{P_3} + N_{asw}(\Delta_3) + N_{aa}(\Delta_2) \right.$

$+ \delta_{su} R_u^{3/2} + \delta_{sa} R_a^{3/2} + \delta_s + \Delta_c + k_h L^3 + \delta_{pe}\left(k_1 L^3 + \beta\right)$

$\left. + \Delta_k \right\} \Big/ \left\{ \delta_f\left[k_4(V^3 L^2)\left(\frac{V_e}{V}\right)^g\right] + \delta_{cs} k_1 L^3 + \delta_{cs_3} \right\}$

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A total cost equation formulated in the manner of (4.9) permits the

military planner to determine what effect varying such system inputs as;

endurance, armament, search capability, cruising speed, and maximum

speed has on the total system cost. The effect of varying certain aspects

of the assigned mission such as; transit distance, size of patrol area,

and revisit time may also be determined from equation (4.9). However,

note that the general nature of the assigned mission cannot be changed

without redefining and restructuring the mission. This inflexibility of

assigned mission structure is perhaps the greatest shortcoming of cost

effectiveness studies, especially when applied to ship systems which are

traditionally multi-purpose weapons systems.

3.   Solutions

A solution of a total cost equation is considered to be the minimum

total system cost within the bounds of the previously fixed degree of

effectiveness and any other constraints initially placed on the weapons

system. Theoretically standard methods for finding minima such as

Lagrange multipliers or simultaneous equations of partial derivatives

could be used to find minima of total cost equations subject to the

desired restraints. However, from the general complexity of a typical

cost equation such as (4.9), it is apparent that an analytical solution

for a minimum point is likely to be an exceedingly difficult and tedious

operation. In addition, it is quite likely that the general shape of the

multi-dimensional surface represented by the general cost equation will
be of more interest to military planners than the location of the minimum
point itself. The type of problem represented here is very well suited to
solution by high speed computers. A typical total cost equation, such
as (4.9), is easy to program on a general purpose digital computer. By
use of computer a large number of values of all of the system inputs can
be investigated as to their effect on the total system cost. Also, a very
good approximation to the minimum cost point can be determined.

4. Extensions

The cost effectiveness study may be easily extended to include
investigations of one weapons systems performance in conjunction with
others. As an example of this type of extension, consider a situation
where it is desired to investigate the relative cost efficiency of operating
the ASW patrol vessels of example "A" with supporting supply ships as
compared to operating the patrol vessels directly from a shore base.

For simplicity's sake assume that the entire resupply of the ASW
patrol ship will be accomplished from one general supply vessel. This
structure can easily be expanded to include the more realistic case where
the resupply mission must be carried out with several ships; tankers,
refrigerator ships, ammunition ships, and the like. Also, assume that
the patrols have been arranged so that the resupply ship will arrive at
each patrol ship at the time the patrol ship is at its low fuel state,
which is generally the limiting factor in endurance. If the patrol vessels

are resupplied at sea, the limiting endurance factor of the patrol vessels is the length of time between necessary repairs that must be performed at a shore base. Denote the repair limited endurance by E_r.

It has been shown in equation (4.1) that N_1 escort vessels must be on station in the patrol area. It now takes

$$(4.12) \qquad N_2 = \frac{E_r + R_2}{E_r - \frac{2d}{24 \, v_e}}$$

patrol vessels to keep one on station continuously, where R_2 represents the required repair and replenishment time per cycle at a shore base. Thus the total number of escorts required is

$$(4.13) \qquad N = \left[\frac{E_r + R_2}{E_r - \frac{2d}{24 \, v_e}} \right] \left[\frac{A}{R \, v_e} \frac{1000}{24 \, T} \right]$$

The number of replenishments required per escort vessel per cycle of $(E_r + R_2)$ days is

$$(4.14) \qquad N_3 = \left[\frac{E_r}{E} - 1 \right] = \left[\frac{E_r - E}{E} \right]$$

and the average number of replenishments required by the system per period of $(E_r + R_2)$ days is $(N) \, (N_3)$.

The number of replenishments a resupply ship can effect during a
period of $(E_r + R_2)$ days, assuming that the resupply ship is supply limited
and not endurance limited, is

$$N_5 = \frac{N4 \, (E_r + R_2)}{R4 + (N4)(R3) + \dfrac{(N4 - 1) \, \ell}{24 \, v_t} + \dfrac{2d}{24 \, v_t}}$$

where: R_4 = repair and replenishment time for resupply ship (days)

ℓ = average distance between replenishment station (miles)

d = average distance to and from the patrol area (miles)

R_3 = average underway replenishment time for escort ship (days)

N_4 = minimum: $\dfrac{F}{f}$, $\dfrac{G}{g}$ where F and f are the fuel capacities

of the replenishment ship and escort ship respectively,

and G and g are similarly the general stores capacities.

It follows that the required number of replenishment ships is found by
dividing N_3 by N_5. The cost elements for supply ships can be structured in
the same manner as has been done for patrol vessels, and a comparison can
then be made of the system efficiency when using unsupported patrol vessels
as opposed to resupplying the patrol vessels at sea.

CHAPTER V

COMMENTS AND CONCLUSIONS

1.   Accuracy

No provisions have been made in the cost effectiveness model presented in this paper for the effect of learning curves on the procurement costs of system units.

No analysis of variance has been conducted on the mathematical expressions of the cost element relationships of Chapter III. An analysis of variance of the cost element relationships would undoubtedly be of great value, and should be done before these relationships are put into actual use.

2.   Conclusions

It appears from this preliminary investigation that cost effectiveness techniques are suitable for use as an aid to selecting optimum characteristics of surface ship weapons systems. As was pointed out previously, the greatest shortcoming of the cost effectiveness study, as applied in this paper, is its inflexibility of mission structure which permits efficiency comparisons only over a rather narrow range of mission assignments without restructuring the model. This limitation is particularly serious in studies concerning warships as a ship is usually an expensive, long life, multi-purpose weapon. However, as long as this limitation is kept in mind, there is no reason why separate mathematical models could not be

formulated to cover all the intended missions, primary and secondary, of

a ship-borne weapons system. This would require separate models only

for the mission part of the total cost equation since the structure of the

individual cost elements would not change as the assigned mission of a

ship was changed. It then remains to decide what weights should be

assigned to the various intended missions of the weapons system. This

assignment of weights to the various intended missions of a weapons

system is no mean task, and must be performed by the military planners

who are to make use of the cost effectiveness study. It should be noted

however, that any time one particular weapons system configuration is

chosen over competing configurations, some sort of weight distribution,

either implicit or explicit, must be made over the various missions to be

performed by the system. Use of the cost effectiveness study as a plan-

ning aid simply requires that this weight distribution be explicit.

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APPENDIX A

SOME ELEMENTARY RELATIONSHIPS CONCERNING ECHO RANGING SONAR

1. List of symbols

R   Range in meters for which the probability of target detection is 50%

a   Sound attenuation coefficient in decibels/meter. For practical

      sonar frequencies $a \cong 10^{-5}(f^2)$

f   Frequency in kilocycles/sec.

A   Attenuation anomoly; sound pressure level loss due to unexplained

      properties of the medium of propigation. When comparing the

      relative merits of two sonars, A is generally considered to be

      zero

ρ Power output of the sonar in watts

γ Target strength; a property of the target surface area

d_r Receiving directivity index; a ratio of the received signal strength

      on the axis of a directional sonar transducer to the theoretical

      signal strength of the same transducer were it omni directional

d_t Transmitting directivity index; similar to d_r except for transmitted

      signals instead of received signals. For echo ranging sonars

      using the same sensitive elements for sending and receiving,

      $d_t = d_r$.

η Ambient noise spectrum level at a frequency of one kilocycle/sec.

h   Vertical dimension in meters of a cylindrical sonar transducer

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c   Velocity of sound in water in meters/sec.

M_n Recognition differential; a measure of the sonar operator's ability to

detect a target over the general background interference

W   Critical band width; a property of the detection system used; i.e.,

electronic, audio, etc.

2.   Active sonar range equation[1]

(A.1) $40 \log R = -2aR - 2A + \tau + 2d_r - \eta + 17 \log f - M_n - 10 \log W$

$$+ 71 + 10 \log (p)^1$$

grouping all the non frequency dependent terms into a constant, ϕ.

$$40 \log r = -2 \times 10^{-5} f^2 r + 20 \log \frac{2hf}{c} + 17 \log(f) + \phi$$

$$\frac{\partial (40 \log r)}{\partial f} = -4 \times 10^{-5} fR + \frac{20}{f} + \frac{17}{f} = 0$$

(A.2) $f_m^2 \cong \dfrac{9 \times 10^5}{R}$

$$\frac{\partial^2 (40 \log r)}{\partial f^2} = -4 \times 10^{-5} R - \frac{20}{f^2} - \frac{17}{f^2} < 0$$

for $R \geqq 0$

[1]Equation (A.1) is developed in unpublished notes of Professor L. E. Kinsler, U. S. Naval Postgraduate School, Monterey, California. Similar results are obtained through a slightly different approach by Horton; J. W. Horton, Fundamentals of Sonar, United States Naval Institute, 1960.

Thus the optimum designed frequency of an echo ranging sonar varies inversely with the square root of its detection range. Using this optimum frequency, the relationships between range, power, and directivity index are

$$40 \log (R) = 18 + 2 d_r + 8.5 \log (9 \times 10^5) - 8.5 \log (R) - 2A + \Upsilon$$
$$- \eta - M_n - 10 \log (W) + 71 + 10 \log (\rho)$$

again by grouping constants we have

(A.3) $\quad \log (R) \cong 0.04 d_r + 0.2 \log(\rho) + \Theta$

From equation (A.3) it is apparent the detection range of an echo ranging sonar is sensitive to the directivity index. Now, let us investigate the relationships involved in determining the value of the directivity index. For a cylindrical transducer of a type currently installed on many ASW vessels

(A.4) $\quad dr \cong 10 \log \dfrac{2hf}{c}$

Thus we see that in order to increase the detection range of an echo ranging sonar it is necessary to either increase the power of the sonar or increase its directivity index. However, if the sonar is to be operated at its optimum frequency, the frequency must be lowered as the range is increased. But lowering the frequency reduces the directivity index unless the size of the transducer is increased. A common design practice used as a solution to this dilemma is to hold the directivity index of a sonar constant while varying the other parameters to increase the range.

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If the directivity index is held constant, and the sonar is designed at its optimum frequency, then from equations (A.2) and (A.4)

(A.5) $$\frac{h^2}{R} = \text{const.}$$

In order to accommodate the greater power necessary for increased ranges it has been necessary to increase the diameter of a cylindrical transducer along with the increase in its vertical dimension in such a manner that the surface area of the transducer is directly proportional to $(h)^2$. Therefore, it is possible to conclude that, under current design practice, the detection range of an echo ranging sonar varies directly with the surface area of its transducer.

It should be pointed out that most of the abore relations are approximations. The quantities W, M_n, and A are not strictly independent of frequency, but their dependence is small enough to be neglected in this development. Also, the equations for a and dr are approximations, but are quite accurate within the normal parameter ranges of modern search sonars. The exact expressions for these terms may be found in standard works on underwater acoustics.

APPENDIX B

THE DEFINITE RANGE LAW OF DETECTION

The definite range law of detection is a simplifying artiface frequently

used in studies concerning search and detection. The definite range law

of detection is derived by taking the range at which the cumulative probability

of detection is equal to 0.50 and postulating that at all lesser ranges the

probability of detection is one, and that at all greater ranges the probability

of detection is zero. For example, suppose the cumulative probability of

detection is of the form

$$P = e^{-b(r)}$$

Then the detection range (R) is such that

$$0.50 = e^{-b(R)}$$

The definite range law of detection is frequently used to obtain com-

parisons between the effectiveness of different search tactics, or the detection

capabilities of different equipments because of its computational simplicity.

It has been shown in various studies that there are certain situations in

which the definite range law of detection will lead to erroneous conclusions.

Consequently, the definite range law of detection should be used with care,

and its use should be carefully scrutinized for possible interjection of bias.

APPENDIX C

DETERMINATION OF THE NUMBER OF ESCORTS REQUIRED TO PROVIDE COMPLETE SONAR COVERAGE ACROSS THE FRONT OF A CONVOY

List of symbols:

v_c Convoy speed (kts.)

v_e Escort speed (kts.)

R   Detection range of escort sonar (50% probability of detection) (yds)

S   Total lateral distance covered by escort sonar search (yds)

x   Lateral distance an escort may move for a given convoy speed, escort speed, and detection range (yds)

r   Lateral distance covered by escort sonar search measured from the extremes of the escort vessel's sweep points (yds)

W   Convoy width, (yds)

Given: A convoy proceeding at speed v_c of width W.

Required: To provide complete sonar coverage across the front of the convoy with surface escort vessels which have sonar detection range R.

By referring to figure 2 it is apparent that if an escort vessel starts at an original relative position 0 with respect to the convoy, and conducts a search pattern by initially moving to the right a distance x to relative position 0', and then returning to its original station, the escort must return to 0 by the time the convoy has traveled a distance 2 λ if the

escort is to provide complete sonar coverage for a distance r to the left

of 0 and to the right of 0'. The time that it will take the convoy to cover

the distance 2λ is

$$t = \frac{2\lambda}{v_c}$$

The lateral distance the destroyer can sweep in this time interval is

$$x = \frac{(v_r)(t)}{2} = \frac{v_r \lambda}{v_c}$$

It follows from figure 2 that

$$\lambda = \sqrt{R_u^2 - r^2}$$
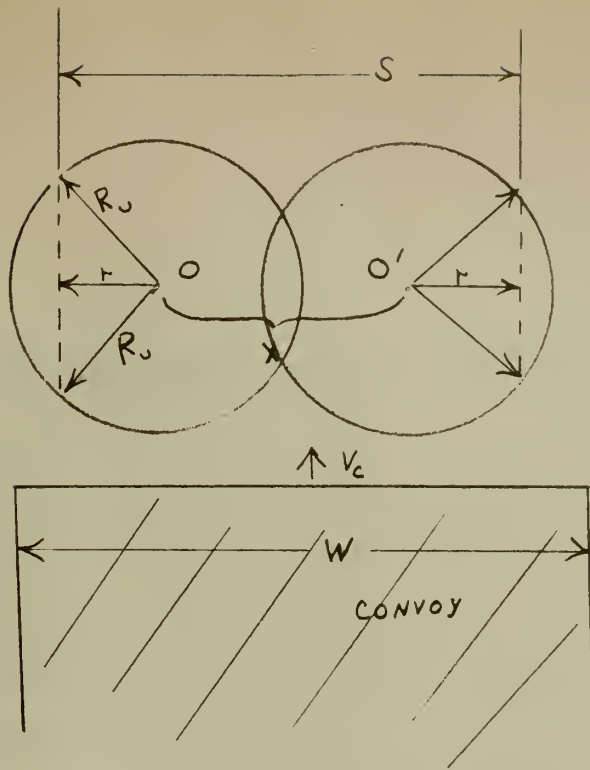
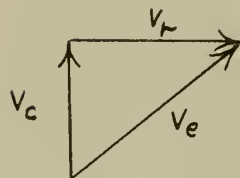and from figure 3 that

$$v_r = \sqrt{v_e^2 - v_c^2}$$

Diagram of Convoy Escort Search Pattern

Figure 2

Vector Diagram of Convoy and Escort Speeds

Figure 3

(C.1) $\dot{X} = \dfrac{V_r}{V_c}\lambda = \left\{\left[\dfrac{V_e^2 - V_c^2}{V_c^2}\right]\left[R_u^2 - r^2\right]\right\}^{1/2}$

The entire width swept by the escort during one cycle is

(C.2) $S = X + 2r = \left\{\left[\dfrac{V_e^2 - V_c^2}{V_c^2}\right]\left[R_u^2 - r^2\right]\right\}^{1/2} + 2r$

Now, to find the value of r that will produce the maximum area

swept per escort at a given speed

$$\dfrac{\partial S}{\partial r} = (-r)\left[\left(\dfrac{V_e^2 - V_c^2}{V_c^2}\right)\left(R_u^2 - r^2\right)\right]^{-1/2}\left(\dfrac{V_e^2 - V_c^2}{V_c^2}\right) + 2 = 0$$

(C.3) $r_m = \dfrac{2R_u}{\left[\dfrac{V_e^2 - 3V_c^2}{V_c^2}\right]^{1/2}}$

$$\dfrac{\partial^2 S}{\partial r^2} = -\dfrac{\left[\dfrac{V_e^2 - V_c^2}{V_c^2}\right]}{\left\{\left[\dfrac{V_e^2 - V_c^2}{V_c^2}\right]\left|R_u^2 - r^2\right|\right\}^{1/2}} - \dfrac{r\left(\dfrac{V_e^2 - V_c^2}{V_c^2}\right)}{\left\{\left[\dfrac{V_e^2 - r^2}{V_c^2}\right]\left[R_u^2 - r^2\right]\right\}^{3/2}} < 0$$

for $r > 0$, $R_u > r$, and $v_e > v_c$

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By substituting (C.3) into (C.2) and letting $\dfrac{V_e}{V_C} = k$

$$S = \left[(k^2-1) \left(R_u{}^2 - \frac{4 R_u{}^2}{|k^2+3|} \right) \right]^{1/2} + \frac{4 R_u}{[k^2+3]^{1/2}}$$

(C.4) $$S = \frac{R_u \sqrt{v_e^2 + 3 v_c^2}}{v_c}$$

The number of escorts required to furnish complete sonar coverage is

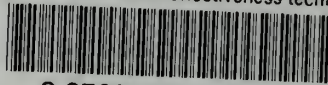(C.5) $$N_1 = \frac{W}{S} = \frac{W v_C}{R_u (v_e^2 + 3 v_C^2)^{1/2}}$$